

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe
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March 16, 2004

QUIZ 1

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USEFUL INFORMATION:

DOPPLER SHIFT:

$$z = v/u \quad (\text{nonrelativistic, source moving})$$

$$z = \frac{v/u}{1 - v/u} \quad (\text{nonrelativistic, observer moving})$$

$$z = \sqrt{\frac{1 + \beta}{1 - \beta}} - 1 \quad (\text{special relativity, with } \beta = v/c)$$

COSMOLOGICAL REDSHIFT:

$$1 + z \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{R(t_{\text{observed}})}{R(t_{\text{emitted}})}$$

EVOLUTION OF A MATTER-DOMINATED UNIVERSE:

$$H^2 = \left(\frac{\dot{R}}{R} \right)^2 = \frac{8\pi}{3} G\rho - \frac{kc^2}{R^2}$$

$$\ddot{R} = -\frac{4\pi}{3} G\rho R$$

$$\rho(t) = \frac{R^3(t_i)}{R^3(t)} \rho(t_i)$$

Flat ($\Omega \equiv \rho/\rho_c = 1$): $R(t) \propto t^{2/3}$

Closed ($\Omega > 1$): $ct = \alpha(\theta - \sin \theta)$,
 $\frac{R}{\sqrt{k}} = \alpha(1 - \cos \theta)$,
where $\alpha \equiv \frac{4\pi}{3} \frac{G\rho R^3}{k^{3/2}c^2}$

Open ($\Omega < 1$): $ct = \alpha(\sinh \theta - \theta)$
 $\frac{R}{\sqrt{\kappa}} = \alpha(\cosh \theta - 1)$,
where $\alpha \equiv \frac{4\pi}{3} \frac{G\rho R^3}{\kappa^{3/2}c^2}$,

$$\kappa \equiv -k .$$

PROBLEM 1: DID YOU DO THE READING? (25 points)

Each of the following parts is worth 5 points. There are blank pages at the back if you need more room for your answers.

- (a) In 1826, the astronomer Heinrich Olber wrote a paper on a paradox regarding the night sky. What is Olber's paradox? What is the primary resolution of it?

- (b) What is the value of the Newtonian gravitational constant G in Planck units? The Planck length is of the order of 10^{-35} m, 10^{-15} m, 10^{15} m, or 10^{35} m?

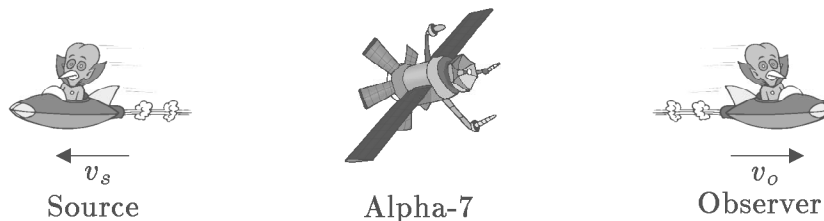
- (c) What is the Cosmological Principle? Is the Hubble expansion of the universe consistent with it? (For the latter question, a simple "yes" or "no" will suffice.)

- (d) In the "Standard Model" of the universe, when the universe cooled to about 3×10^a K, it became transparent to photons, and today we observe these as the Cosmic Microwave Background (CMB) at a temperature of about 3×10^b K. What are the integers a and b ?

- (e) What did the universe primarily consist of at about 1/100th of a second after the Big Bang? Include any constituent that is believed to have made up more than 1% of the mass density of the universe.

PROBLEM 2: SPECIAL RELATIVITY DOPPLER SHIFT (20 points)

Consider the Doppler shift of radio waves, for a case in which both the source and the observer are moving. Suppose the source is a spaceship moving with a speed v_s relative to the space station Alpha-7, while the observer is on another spaceship, moving in the opposite direction from Alpha-7 with speed v_o relative to Alpha-7.



- (a) (10 points) Calculate the Doppler shift z of the radio wave as received by the observer. (Recall that radio waves are electromagnetic waves, just like light except that the wavelength is longer.)
- (b) (10 points) Use the results of part (a) to determine v_{tot} , the velocity of the source spaceship as it would be measured by the observer spaceship. (8 points will be given for the basic idea, whether or not you have the right answer for part (a), and 2 points will be given for the algebra.)

PROBLEM 3: EVOLUTION OF A FLAT UNIVERSE WITH $R(t) = bt^{1/4}$ (35 points)

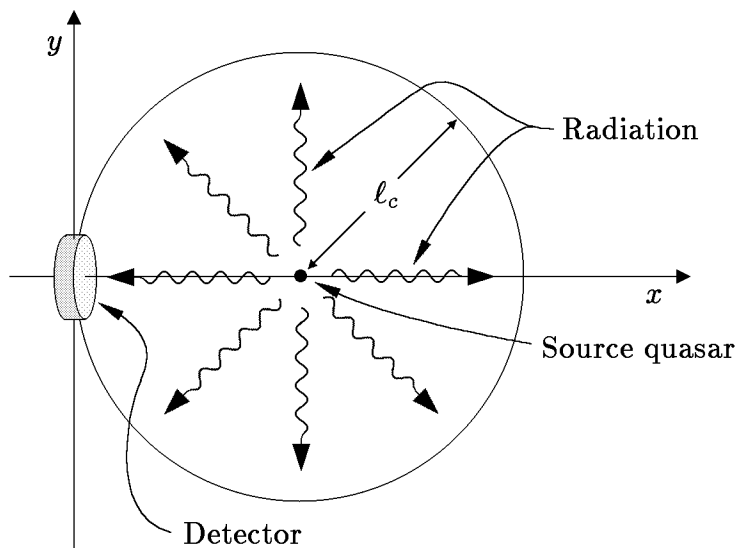
The following questions all pertain to a flat universe, with a scale factor given by

$$R(t) = bt^{1/4},$$

where b is a constant and t denotes cosmic time.

- (a) (10 points) If a light pulse is emitted by a quasar at time t_e and observed here on Earth at time t_o , find the physical separation $\ell_p(t_e)$ between the quasar and the Earth, at the time of **emission**.
- (b) (10 points) At what time is the light pulse equidistant from the quasar and the Earth?
- (c) (15 points) At the time of emission, the quasar had a power output P (measured, say, in ergs/sec), which was radiated uniformly in all directions. What is the radiation energy flux J from this quasar at the Earth today? Energy flux (which might be measured in ergs-cm⁻²-sec⁻¹) is defined as the energy per unit area per unit time striking a surface that is orthogonal to the direction of

energy flow. You may find it useful to think of the detector as a small part of a sphere that is centered on the source, as shown in the following diagram:



PROBLEM 4: EVOLUTION OF A CLOSED, MATTER-DOMINATED UNIVERSE (20 points)

The following problem was Problem Set 2, Problem 4.

It was shown in Lecture Notes 5 that the evolution of a closed, matter-dominated universe can be described by introducing the time-parameter θ , with

$$ct = \alpha(\theta - \sin \theta) ,$$

$$\frac{R}{\sqrt{k}} = \alpha(1 - \cos \theta) ,$$

where α is a constant with the units of length.

- (10 points) Use these expressions to find H , the Hubble “constant,” as a function of α and θ . (Hint: You can use the first of the equations above to calculate $d\theta/dt$.)
- (5 points) Find ρ , the mass density, as a function of α and θ . (4 points will be given for a correct answer, with 1 additional point if the answer is algebraically simplified.)
- (5 points) Find Ω , where $\Omega \equiv \rho/\rho_c$, as a function of α and θ .