MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe — March 3, 1998

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QUIZ 1

USEFUL INFORMATION:

COSMOLOGICAL REDSHIFT:

$$1 + z \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \frac{R(t_{\text{observed}})}{R(t_{\text{emitted}})}$$

NOTE: Any answer may be expressed in terms of symbols representing the answers to previous parts of the same question, whether or not the previous part was answered correctly. When I say that an answer should be expressed in terms of a specified set of variables, I mean that the variables used in your answer should be drawn from that set; this does not imply that all the variables in the set need be used.

PROBLEM 1: DID YOU DO THE READING? *(30 points)*

The following questions are worth 5 points each.

a) Name the two men who in 1964 discovered the cosmic background radiation. With what institution were they affiliated?

b) In 1917, Einstein introduced a model of the universe which was based on his newly developed general relativity, but which contained an extra term in the equations which he called the "cosmological term." (The coefficient of this term is called the "cosmological constant.") What was Einstein's motivation for introducing this term?

c) When the redshift of distant galaxies was first discovered, the earliest observations were analyzed according to a cosmological model invented by the Dutch astronmer W. de Sitter in 1917. At the time of its discovery, was this model thought to be static or expanding? From the modern perspective, is the model thought to be static or expanding?

d) The early universe is believed to have been filled with thermal, or black-body, radiation. For such radiation the number density of photons and the energy density are each proportional to powers of the absolute temperature T. Say

$$\text{Number density} \propto T^{n_1}$$

$$\text{Energy density} \propto T^{n_2}$$

Give the values of the exponents n_1 and n_2.

e) Some of the earliest measurements of the cosmic background radiation were made indirectly, by observing interstellar clouds of a molecule called cyanogen (CN). State whether each of the following statements is true or false *(1 point each)*:

(i) The first measurements of the temperature of the interstellar cyanogen were made over twenty years before the cosmic background radiation was directly observed.

(ii) Cyanogen helps to measure the cosmic background radiation by reflecting it toward the earth, so that it can be received with microwave detectors.

(iii) One reason why the cyanogen observations were important was that they gave the first measurements of the equivalent temperature of the cosmic background radiation at wavelengths shorter than the peak of the black-body spectrum.

(iv) By measuring the spectrum of visible starlight that passes through the cyanogen clouds, astronomers can infer the intensity of the microwave radiation that bathes the clouds.

(v) By observing chemical reactions in the cyanogen clouds, astronomers can infer the temperature of the microwave radiation in which they are bathed.

f) At about 3,000 K the matter in the universe underwent a certain chemical change in its form, a change that was necessary to allow the differentiation of matter into galaxies and stars. What was the nature of this change?

PROBLEM 2: ANOTHER FLAT UNIVERSE WITH AN UNUSUAL TIME EVOLUTION *(30 points)*

The following problem was Problem 4, Quiz 2, 1992, and was included with the Review Problems for this year:

Consider a ***flat*** universe which is filled with some peculiar form of matter, so that the Robertson–Walker scale factor behaves as

$$R(t) = bt^{\gamma},$$

where b and γ are constants. *[This universe differs from the matter-dominated universe described in the lecture notes in that ρ is not proportional to $1/R^3(t)$. Such behavior is possible when pressures are large, because a gas expanding under pressure can loose energy (and hence mass) during the expansion.]* For the following questions, any of the answers may depend on γ, whether it is mentioned explicitly or not.

a) *(8 points)* Let t_0 denote the present time, and let t_e denote the time at which the light that we are currently receiving was emitted by a distant object. In terms of these quantities, find the value of the redshift parameter z with which the light is received.

b) *(4 points)* Find the "look-back" time as a function of z and t_0. The look-back time is defined as the length of the interval in cosmic time between the emission and observation of the light.

c) *(8 points)* Express the present value of the physical distance to the object as a function of H_0, z, and γ.

d) *(10 points)* At the time of emission, the distant object had a power output P (measured, say, in ergs/sec) which was radiated uniformly in all directions, in the form of photons. What is the radiation energy flux J from this object at the earth today? Express your answer in terms of P, H_0, z, and γ. *[Energy flux (which might be measured in erg-cm^{-2}-sec^{-1}) is defined as the energy per unit area per unit time striking a surface that is orthogonal to the direction of energy flow.]*

PROBLEM 3: A FLAT UNIVERSE WITH $R(t) \propto t^{3/5}$ *(40 points)*

Consider a ***flat*** universe which is filled with some peculiar form of matter, so that the Robertson–Walker scale factor behaves as

$$R(t) = bt^{3/5} ,$$

where b is a constant.

a) *(5 points)* Find the Hubble constant H at an arbitrary time t.

b) *(10 points)* Suppose a message is transmitted by radio signal (traveling at the speed of light c) from galaxy A to galaxy B. The message is sent at cosmic time t_1, when the physical distance between the galaxies is ℓ_0. At what cosmic time t_2 is the message received at galaxy B? (Express your answer in terms of ℓ_0, t_1, and c.)

c) *(5 points)* Upon receipt of the message, the creatures on galaxy B immediately send back an acknowledgement, by radio signal, that the message has been received. At what cosmic time t_3 is the acknowledgment received on galaxy A? (Express your answer in terms of ℓ_0, t_1, t_2, and c.)

d) *(10 points)* The creatures on galaxy B spend some time trying to decode the message, finally deciding that it is an advertisement for Kellogg's Corn Flakes (whatever that is). At a time Δt after the receipt of the message, as measured on their clocks, they send back a response, requesting further explanation. At what cosmic time t_4 is the response received on galaxy A? In answering this part, you should not assume that Δt is necessarily small. (Express your answer in terms of ℓ_0, t_1, t_2, t_3, Δt, and c.)

e) *(5 points)* When the response is received by galaxy A, the radio waves will be red-shifted by a factor $1 + z$. Give an expression for z. (Express your answer in terms of ℓ_0, t_1, t_2, t_3, t_4, Δt, and c.)

f) *(5 points; No partial credit)* If the time Δt introduced in part (d) is small, the time difference $t_4 - t_3$ can be expanded to first order in Δt. Calculate $t_4 - t_3$ to first order accuracy in Δt. (Express your answer in terms of ℓ_0, t_1, t_2, t_3, t_4, Δt, and c.) [Hint: while this part can be answered by using brute force to expand the answer in part (d), there is an easier way.]