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THE EFFECT OF URANIUM-236 AND NEPTUNIUM-237 ON THE VALUE OF URANIUM USED AS FEED FOR THERMAL POWER REACTORS

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FOR UNITED STATES ATOMIC ENERGY COMMISSION
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The calculational methods used in this report were developed by Donald A. Goellner in an investigation of a pressurized water reactor which yielded fuel cycle characteristics utilized in this report. Dr. Goellner's assistance throughout this work was invaluable.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

I. INTRODUCTION

Report MIT-2073-6 (1) described a method for calculating the effect of uranium-236 and neptunium-237 on the value of uranium and applied the method to uranium used as feed for a pressurized water reactor. The present report extends application of the method to a heavy water reactor and to a pressurized water reactor whose spent uranium is then fed to a heavy water reactor.

As the optimum U-235 content of feed for a heavy water reactor is between natural uranium and 1.5%, whereas the optimum for a pressurized water reactor is between 2 and 3%, the present results are of interest because they extend the range of enrichments over which uranium containing U-236 has been valued by this method.

The present report is also of interest because it avoids a complication in the evaluation procedure used in the previous report which arose in determining the value of spent uranium discharged from a light water reactor. This uranium contains around 1% U-235 and some U-236 and has sufficient value to require consideration in the economic analysis. The fuel cycle cost equation for such a reactor when fed with uranium containing U-236 contains two unknowns, the value of feed uranium and the value of spent uranium, and thus does not permit unique determination of the value of feed uranium. In the previous report this difficulty was

dealt with by assuming that the spent uranium would be recycled to provide part of the feed for the reactor, either by reenriching the spent uranium in a diffusion plant or by blending it with more highly enriched uranium.

This complication is not encountered in the present report. Spent uranium from a heavy water reactor typically contains so little U-235 that it may be assigned zero value; the value of uranium containing U-236 when used as feed for a heavy water reactor then may be determined uniquely from the fuel cycle cost equation.

For the light water reactor also, the present report avoids the assumption that spent uranium would be recycled. Spent uranium from a light water reactor typically contains around 1% U-235 and some U-236, and is in the composition range of uranium whose value has been determined when used as feed in a heavy water reactor. By making use of this fact, the value of uranium feed to a light water reactor is determined from its fuel cycle cost equation, with spent uranium assigned the value it would have if used as feed in a heavy water reactor.

The following principle was used to determine the value of uranium whose composition was specified as weight ratio R of U-235 to U-238 and weight fraction y of U-236. For a given reactor, with a given fuel cycle flowsheet, fueled with uranium free from U-236 and valued on the AEC's price

scale for uranium as a function of enrichment, fuel cycle costs were determined for a series of feed enrichments to find the optimum enrichment and the corresponding minimum fuel cycle cost. Uranium of U-235 to U-238 weight ratio R and U-236 weight fraction y was then assumed fed to the same reactor using the same fuel cycle flowsheet, and this feed was given that value which led to the same minimum fuel cycle cost as uranium containing no U-236 of optimum enrichment valued on the AEC price scale. If uranium containing U-236 could be bought for this value, it would be a matter of indifference to the reactor operator whether he bought this uranium or uranium free from U-236 at the AEC's prices.

The AEC price scale used in the present work (2) is the one in effect from July 1962 through December 1967, based on a charge of $30/kg for separative work. On January 1, 1968 (3), the charge for separative work was reduced to $26/kg. This revision in the price scale would reduce all uranium values given in this report, but would have little effect on the difference between values for uranium determined in this work and the AEC's prices for uranium of the same R.

The principal economic parameters used in the present report are listed in Table I.1.

TABLE I.1

Economic Parameters

Reactor	HWR	PWR
U_3O_8 Price ($/lb), $C_{U_3O_8}$	6,8,10	8
Np-237 Price ($/g Np), C_N	0,20,60,100	0,60
Fissile Plutonium Price ($/g fissile Pu)*, C_K	9.01,10.00 10.94	10.00
Separative Work ($/kg U), C_Δ	30.00	30.00
Fabrication Cost ($/kg U)(includes shipping), C_F	40.00	60.00
Reprocessing Cost ($/kg U), C_A	25.00	40.00
Spent Fuel Shipping ($/kg U), C_{SH}	3.00	6.00
Fuel Storage, in lieu of recovery, C_{STOR}	3.00	
Fractional Loss During Reprocessing (Plutonium), L_{RP}***	0.01	0.01
(Uranium), L_{RU}***	0.01	0.01
Fractional Loss During Fabrication, L_F**	0.01	0.01
Pre-Irradiation Holdup Time (years), t_F	0.2	0.356

TABLE I.1 (Continued)

Reactor	HWR	PWR
Post-Irradiation Holdup Time (years) (Plutonium), t_{RP}	0.67	0.548
(Uranium), t_{RU}		0.603
Annual Charge on Fuel Inventory (year^{-1}), i	0.10	0.10
Load Factor, L	0.80	0.80
Cost of Converting UO_3 to UF_6 ($/kg U), C_C	4.00	4.00
Cost of Converting UO_3 to UF_6 (including shipping cost)^{6}($/kg U), C_{CT}	5.00	5.00
UO_3 to UF_6 Conversion Holdup Time (years), t_C	0.082	0.082
Toll Enrichment Holdup Time (years), t_E	0.25	0.25
Fractional Loss During Conversion, L_C	0.003	0.003

* 10/12 the price of U-235, 90% enriched, as nitrate

** based on fuel leaving fabrication plant

*** based on material entering reprocessing plant

The alternative values for natural uranium of $6, 8 or 10 per pound U_3O_8 cover the range anticipated for the next decade. The alternative values of $0, 20, 60 or 100 per gram Np-237 cover the range of prices which will probably be offered for this material when a market develops for it as a target material for production of Pu-238. These neptunium prices are for material in fuel as discharged from the reactor and do not include recovery costs.

The alternative plutonium prices of $9.01, 10.00 and 10.94 per gm correspond to natural uranium prices of $6, 8 or 10 per lb U_3O_8 and have been computed as 10/12 the price of a gram of U-235 at 90% enrichment on the AEC price scale. Equations for the"AEC price scale" are given in Appendix A. As this work was carried out in 1967, the charge for separative work then prevailing, $30/kg, was used.

Unit prices for fuel cycle operations for the heavy water reactor were selected after considering those used by Atomics International and Combustion Engineering in designing the Heavy Water Organic Cooled Reactor (HWOCR) (4) and those used by Oak Ridge National Laboratory (5) in evaluating this reactor. Fuel cycle cost parameters for the pressurized water reactor are the "high-cost, high-loss" value of the previous report (1) on this project.

The heavy water reactor used for this study is a 1073 Mwe uranium carbide-fueled, organic-cooled, heavy water-moderated reactor (HWOCR) similar in all essential

respects to the one designed by Atomics International and Combustion Engineering (4). Details of this reactor are given in section III and Appendix B. Results for the effect of U-236 on uranium value obtained for this HWOCR are considered representative of large heavy water reactors designed for good neutron economy; hence, they are characterized as applying to heavy water reactors (HWR) as a class.

The pressurized water reactor (PWR) is the 430 Mwe San Onofre reactor designed by Westinghouse for Southern California Edison Co. and San Diego Gas and Electric Co. Fuel design details were obtained from the Westinghouse design report (6), except that 24.3 mil zircaloy cladding was assumed in place of the stainless steel cladding specified by Westinghouse. Its principal characteristics are summarized in Table B2 of Appendix B.

All cost calculations in this report and all uranium values refer to a reactor fuel cycle which has reached a steady state with respect to U-236 and Np-237 concentrations. In practical recycle operations U-236 and Np-237 concentrations build up gradually, and steady-state concentrations aren't reached for a number of years.

II. SUMMARY

A. Heavy Water Reactor

When the heavy water reactor is fed with uranium free from U-236 priced on the AEC price scale, the optimum weight ratio of U-235 to U-238 in feed R^*, which results in minimum fuel cycle cost C_E^*, depends on the cost of natural uranium $C_{U_3O_8}$ and the unit credit for neptunium C_N as summarized below.

$C_{U_3O_8}$, $/lb	C_N, $/g Np	Optimum wt. Ratio, R^*	Minimum Fuel Cycle Cost C_E^*, mills/kwh
10	0	0.01299	0.8584
10	60	0.01317	0.8240
8	0	0.01351	0.7890
8	60	0.01368	0.7542
6	0	0.01408	0.7165
6	60	0.01423	0.6578

Additional results are given in Table IV.3. The optimum enrichment increases with decreasing U_3O_8 price and increasing neptunium price. The minimum fuel cycle cost decreases with decreasing U_3O_8 price and increasing neptunium price, as would be expected.

When the heavy water reactor is fed with uranium containing y weight fraction U-236 and R weight ratio of U-235 to U-238 (possibly different from R^*), the value of this feed V(R,y) has been determined from the valuational principle that the fuel cycle cost shall equal the minimum cost C_E^* obtainable when uranium free of U-236, of optimum enrichment R^*, is purchased on the AEC price scale. Fuel values for twelve

combinations of natural U_3O_8 prices $C_{U_3O_8}$ of \$6, 8 and 10/lb and neptunium prices C_N of \$0, 20, 60 and \$100/g are given in Table VII.1.

Figure II.1 shows the dependence of uranium value on R and y for $C_{U_3O_8}$ = \$8/lb and C_N = \$0/g. Uranium values increase with increasing R and decrease with increasing U-236 content. When uranium contains no U-236 (y = 0), the value curve is tangent to the line representing the AEC price scale at the optimum R value R^*, and lies below the AEC price scale at all other values of R. This is a necessary consequence of the valuation principle.

Figure II.2 is a similar plot for $C_{U_3O_8}$ = \$8/lb and C_N = \$60/g. This differs from Figure II.1 in that the uranium value now <u>increases</u> with increasing U-236 content, at least above R = 0.0104. When the neptunium price is as high as \$60/g, the additional neptunium produced in the reactor when U-236 is present in the feed decreases fuel cycle costs more than the increase in fuel cycle cost resulting from the poisoning effect of the U-236.

Material flow for this so-called base case, in which uranium feed whose value V(R,y) is to be determined is fed directly to the reactor, is shown in Figure II.3a.

Figure II.1 shows that when the uranium enrichment of reactor feed departs appreciably from the optimum enrichment R*, its base case value, when C_N = \$0/g, drops substantially below the AEC price for uranium. This indicates that it is uneconomic to

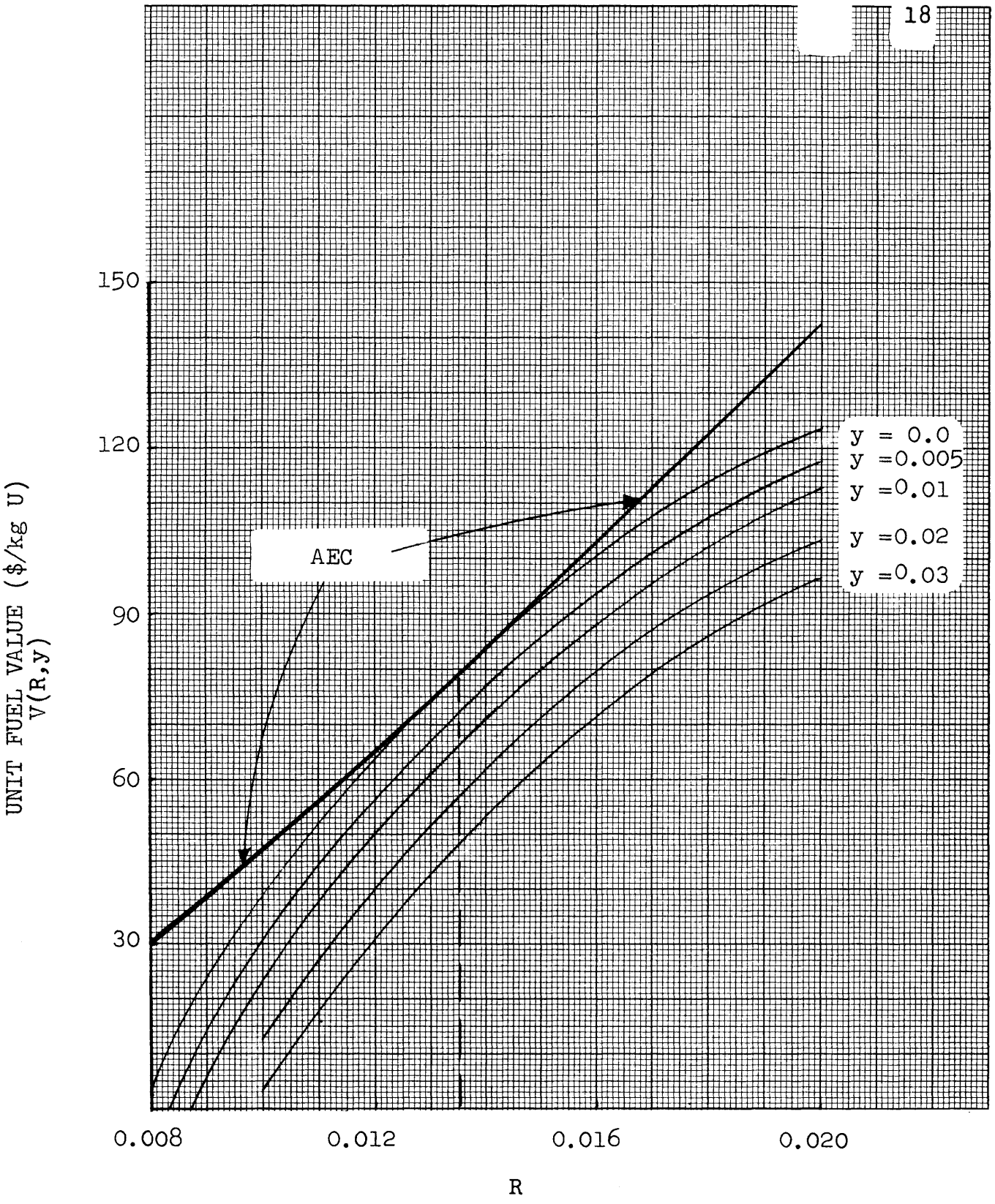


FIGURE II.1 The Effect of R and y on Base Case Unit Fuel Value - $C_{U_3O_8}$ = \$8/lb, C_N = \$0/g. HWR.

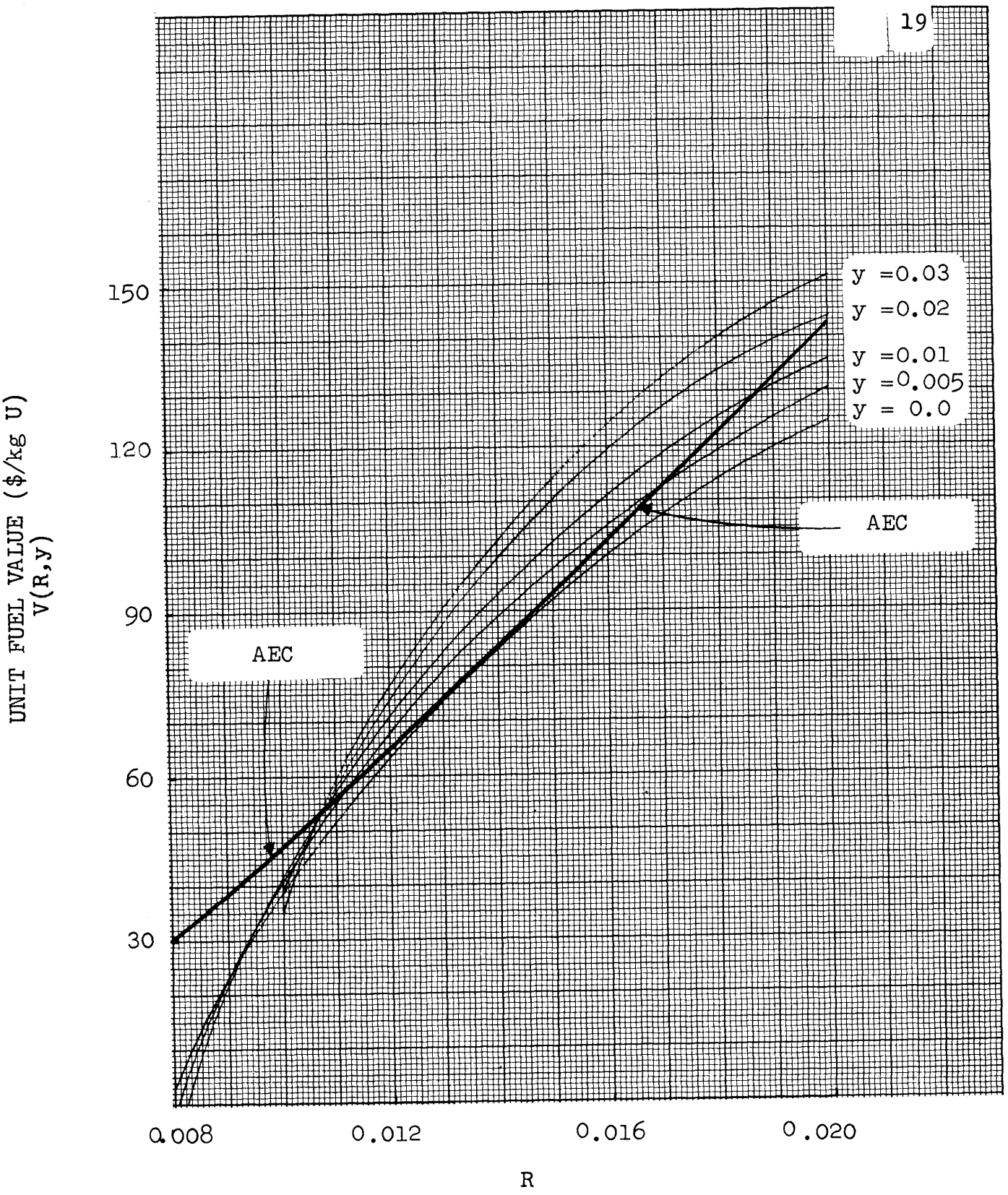
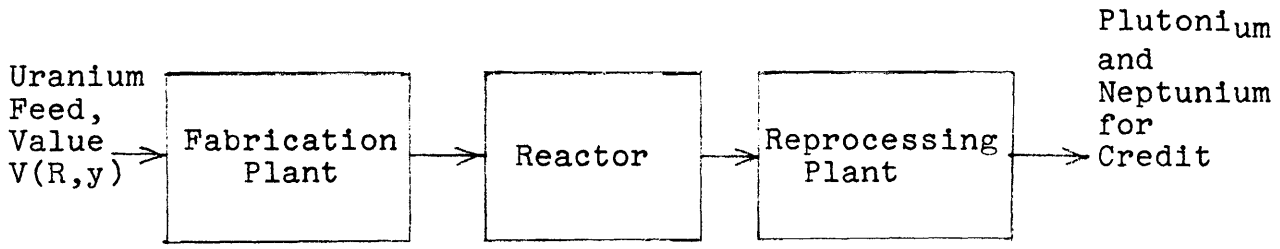
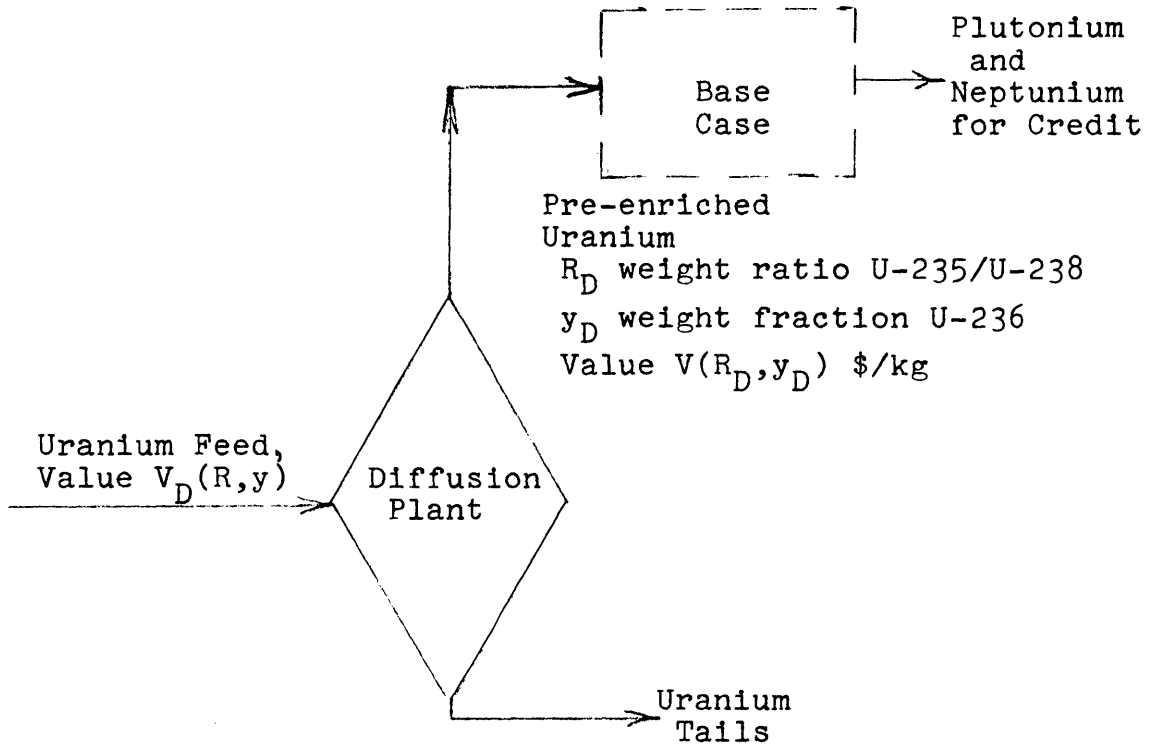


FIGURE II.2 The Effect of R and y on Base Case Unit Fuel Value - $C_{U_3O_8}$ = \$8/lb, C_N = \$60/g . HWR.

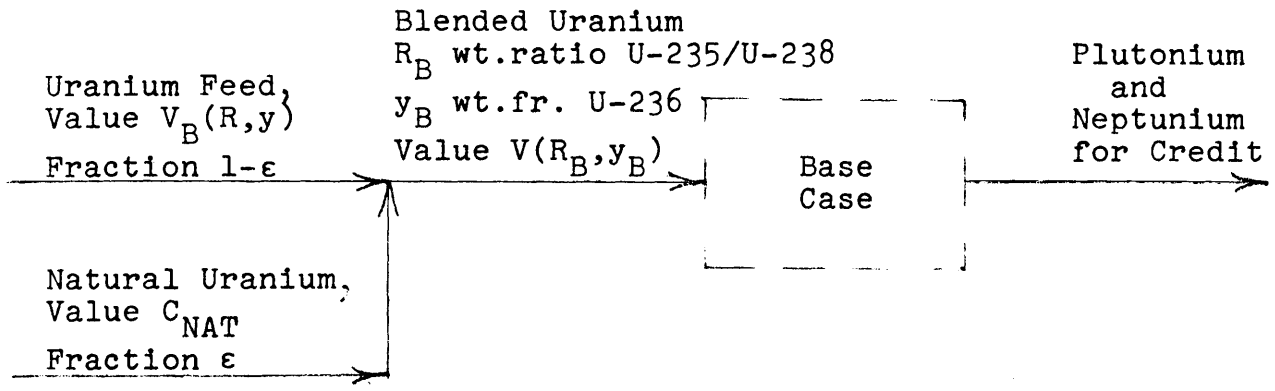
FIGURE II.3 Material Flow for Base Case and Two Modified Cases



a. Base Case



b. Pre-enrichment by Gaseous Diffusion



c. Blending with Natural Uranium

feed uranium of such non-optimum enrichment directly to the reactor, and requires that modified flow-scheme cases be investigated.

When the enrichment of uranium is well below the optimum R^*, higher fuel values are obtainable by preenriching the uranium in a gaseous diffusion plant as illustrated in Figure II.3b, to an optimum enrichment R_D which leads to maximum fuel value $V_D(R,y)$. When the enrichment of uranium is well above the optimum R^*, higher fuel values are obtainable by blending the uranium with natural uranium as illustrated in Figure II.3c. The blending fraction of natural uranium ε which leads to maximum fuel value $V_B(R,y)$ is determined.

Figure II.4 shows how the three fuel values, for the base case $V(R,y)$, for preenrichment by gaseous diffusion $V_D(R,y)$ and for blending with natural uranium $V_B(R,y)$, vary with uranium enrichment R and U-236 weight fraction y for $C_{U_3O_8}$ = \$8/lb and C_N = \$0/g. Figure II.5 gives similar information for C_N = \$60/g. The improvement in fuel values from using the modified cases, especially at enrichments far from R^*, are notable.

Highest values of V, V_D and V_B at each uranium composition R,y investigated are called maximum uranium values $V_m(R,y)$. Maximum uranium values for seven combinations of $C_{U_3O_8}$ and C_N and the corresponding optimum mode of operation are tabulated in Table VIII.1.

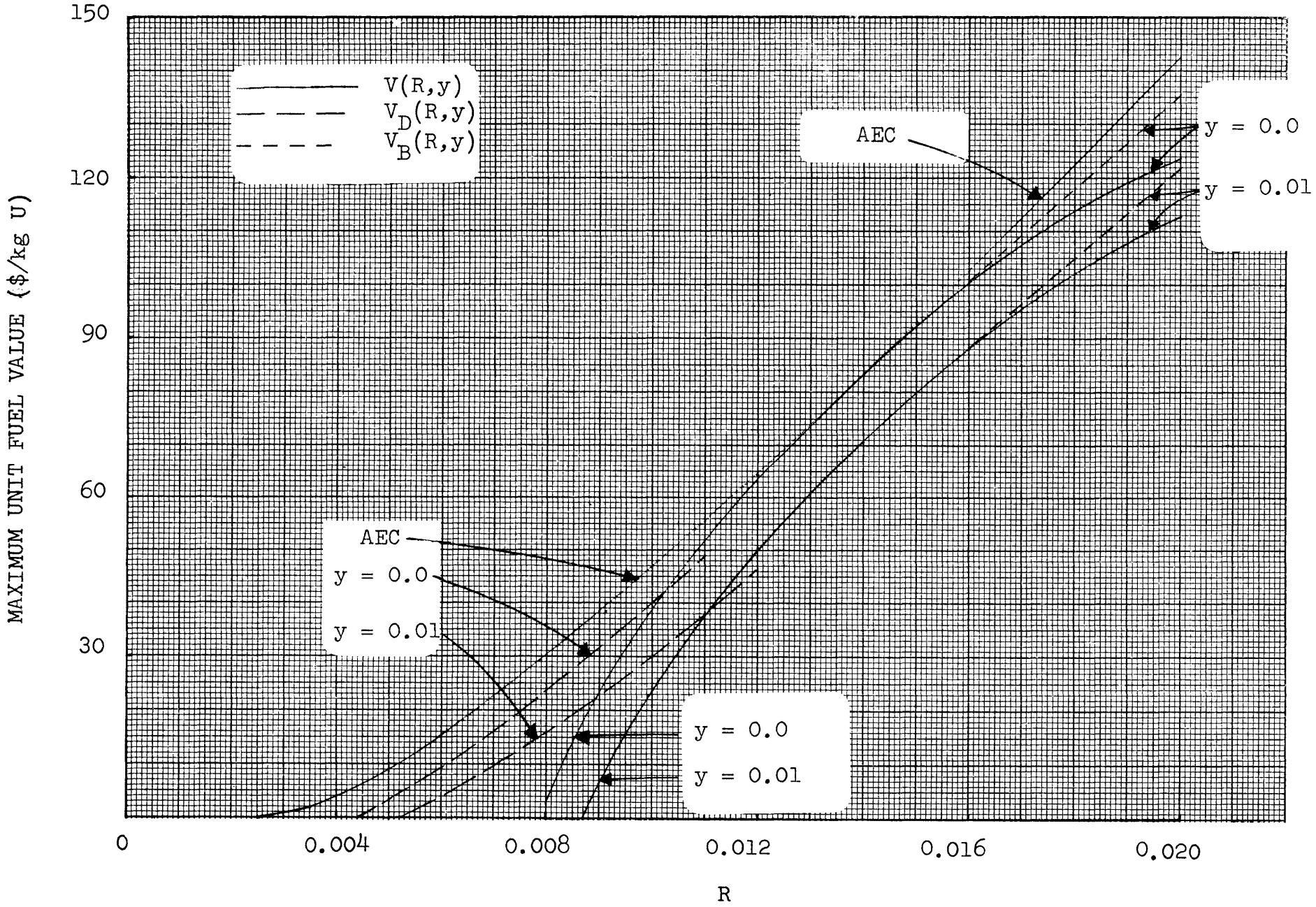


FIGURE II.4 The Effect of R and y on the Maximum Unit Fuel Value - $C_{U_3O_8}$ = \$8/lb, C_N = \$0/g. HWR.

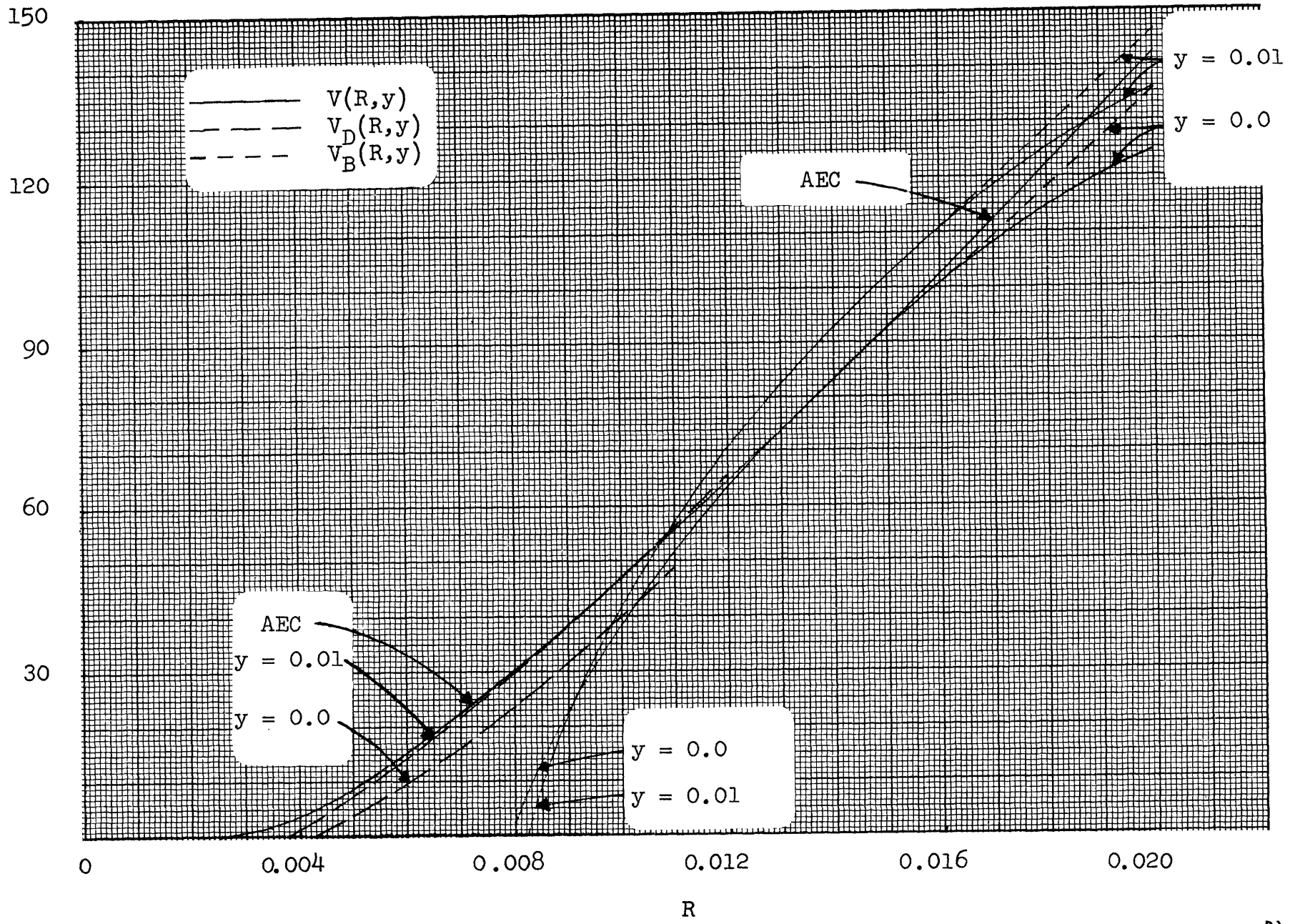


FIGURE II.5 The Effect of R and y on the Maximum Unit Fuel Value - $C_{U_3O_8}$ = \$8/lb, C_N = \$60/g. HWR.

The designer or operator of a reactor will usually know values of V_m for uranium containing no U-236, $V_m(R,0)$, but may not have determined values for uranium containing U-236. To facilitate calculation of the value of uranium containing U-236 from the value of uranium of the same U-235 to U-238 ratio free from U-236, a U-236 penalty, δ, has been evaluated. This penalty is defined by the equation

$$\delta(R,y) = \frac{(1-y)V_m(R,0) - V_m(R,y)}{1000y} \qquad \text{(II.1)}$$

δ has the units of \$/g U-236. It is the reduction in value of (1-y) kg of uranium containing R weight ratio of U-235 to U-238 when y kg of U-236 are added, per gram of U-236 added. Figure II.6 shows how this U-236 penalty varies with uranium enrichment, natural uranium price $C_{U_3O_8}$ and neptunium price C_N, for uranium containing 0.01 weight fraction U-236. The irregular character of these curves is due to the change from one mode of operation to another as R changes, as explained in more detail in section VIII. Table VIII.2 shows that the U-236 penalty decreases slightly as the U-236 content of uranium increases.

Figure II.6 shows that the U-236 penalty is positive at a neptunium price C_N of 0, but becomes negative when C_N = \$60/g. As explained earlier, at this neptunium price the credit for the additional neptunium produced when U-236 is added to reactor feed is greater than the cost penalty caused by the poisoning effect of the U-236. From these results a

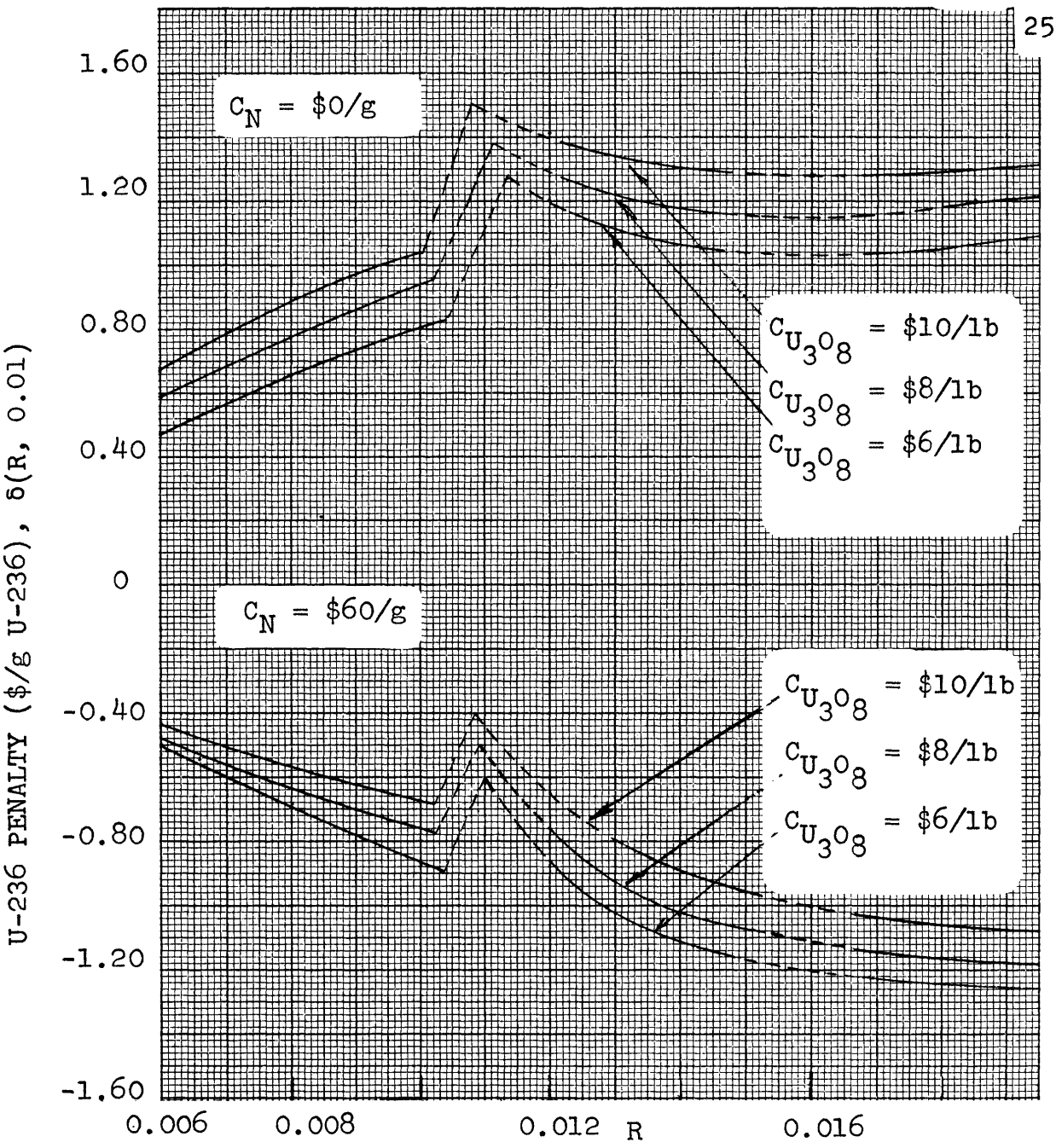


FIGURE II.6 The Effect of U_3O_8 and Np-237 Price on the U-236 Penalty - HWR
y = 0.01

neptunium "indifference price" has been evaluated, at which addition of U-236 to uranium would have no effect on its value as feed for this reactor. This indifference price ranges from \$28.40/g at $C_{U_3O_8}$ = \$6/lb and y = 0.005 to \$37.10/g at $C_{U_3O_8}$ = \$10/g and y = 0.03.

B. Pressurized Water Reactor

As explained in the Introduction, the spent uranium discharged from the pressurized water reactor considered in this report was assigned the maximum value it would have if used as feed to a heavy water reactor, determined as explained in section A, above.

The following tabulation compares the minimum fuel cycle cost C_E^* in the pressurized water reactor when spent fuel is credited at the value it would have as feed for a heavy water reactor with the minimum fuel cycle cost found in the previous report (1) when spent fuel is recycled through a diffusion plant. These fuel cycle costs assume that feed to the PWR contains no U-236 and is valued on the AEC's price scale. The optimum weight ratio of U-235 to U-238 is also given.

		Recycle through Diffusion Plant		Spent U to HWR	
$C_{U_3O_8}$	C_N	R^*	C_E^*	R^*	C_E^*
(\$/lb)	(\$/g Np)		(mills/kwh)		(mills/kwh)
8	0	0.0309	1.614	0.0315	1.526
8	60	0.0315	1.429	0.0320	1.430

The way in which spent fuel is treated has little effect on the optimum enrichment, but it is noteworthy that the minimum fuel cycle cost is almost 0.1 mills/kwh lower when spent fuel is fed to an HWR than when it is recycled through a diffusion plant, when neptunium has no value. This is because U-236 from spent fuel is concentrated in the diffusion plant product and is returned to and poisons the PWR, whereas it is not returned when spent fuel is fed to an HWR. When neptunium is priced at $60/g, there is little difference between the minimum fuel cycle costs, because the credit for the additional neptunium made when spent uranium is recycled through the diffusion plant about offsets the poisoning effect of the U-236.

The value of uranium containing U-236 when used as feed to a PWR whose spent fuel is credited at the value it would have if fed to an HWR was then determined from the principle that the PWR feed should have that value which made the fuel cycle cost for the PWR the same as the minimum fuel cycle cost discussed in the previous paragraph. Uranium values were determined in this way for the PWR, for the base case mode of operation and for the two modified modes illustrated in Figure II.1, preenrichment by gaseous diffusion and blending with natural uranium. Maximum fuel values at a number of values of R and y are given in Table II.1, together with the mode of operation which led to the maximum values.

TABLE II.1

Maximum Unit Fuel Values in PWR, with Spent Uranium Credited as Fuel in HWR, $/kg U

R =	0.015	0.020	0.025	0.030	0.04	0.05	0.06	0.08	1.0	15.0
	$C_{U_3O_8}$ = \$8/lb; C_N = \$0/g									
y = 0.00	84.052	131.63	(187.00)	(244.02)	340.03	432.49	523.21	699.62	4999.0	9413.4
0.01	61.657	105.22	151.15	(209.13)	306.22	397.10	486.50	660.64	4916.2	9285.5
0.025				(170.77)	266.17	352.41	438.40	607.46	4792.6	9094.2
	$C_{U_3O_8}$ = \$8/lb; C_N = \$60/g									
y = 0.00	84.023	131.59	180.60	(243.91)	340.29	432.84	523.64	700.19	5004.0	9418.1
0.01	92.312	140.89	(191.34)	(253.52)	352.24	444.09	534.12	709.07	4970.4	9339.9
0.025			(195.39)	(262.82)	365.94	457.76	547.27	720.55	4919.7	9222.4

Values in parentheses are from base case. Values at lower R are from pre-enrichment by gaseous diffusion. Values at higher R are from blending with natural uranium.

The middle, solid line of Figure II.7 shows the variation with R of the maximum value of uranium containing 1 w/o U-236 when used as feed in a PWR whose spent uranium is credited with the value it would have as feed in a PWR, for zero neptunium credit and for natural uranium priced at \$8/lb U_3O_8. The lower, broken line is the corresponding maximum value of uranium when used as feed to a PWR whose spent uranium is recycled back to the reactor through a diffusion plant. Under these conditions, uranium value is about \$60/kg higher when spent fuel is sent to an HWR than when it is recycled through a diffusion plant. This, of course, is because U-236 doesn't build up in the reactor in the first instance. The difference between uranium values for these two ways of dealing with spent fuel are much less at a neptunium value of \$60/g.

U-236 penalties, defined as in Eq. (II.1) for uranium fed to a PWR with spent uranium sent to an HWR are plotted in Figure II.8. The absolute magnitude of these penalties is greater than those in the HWR shown in Figure II.6, but is only about 30% of that in a PWR whose spent uranium is recycled through a diffusion plant (1). This, again, is because of the buildup of U-236 when uranium is recycled through a diffusion plant.

The neptunium price at which the penalty would be zero, the so-called indifference value, is around \$44/g.

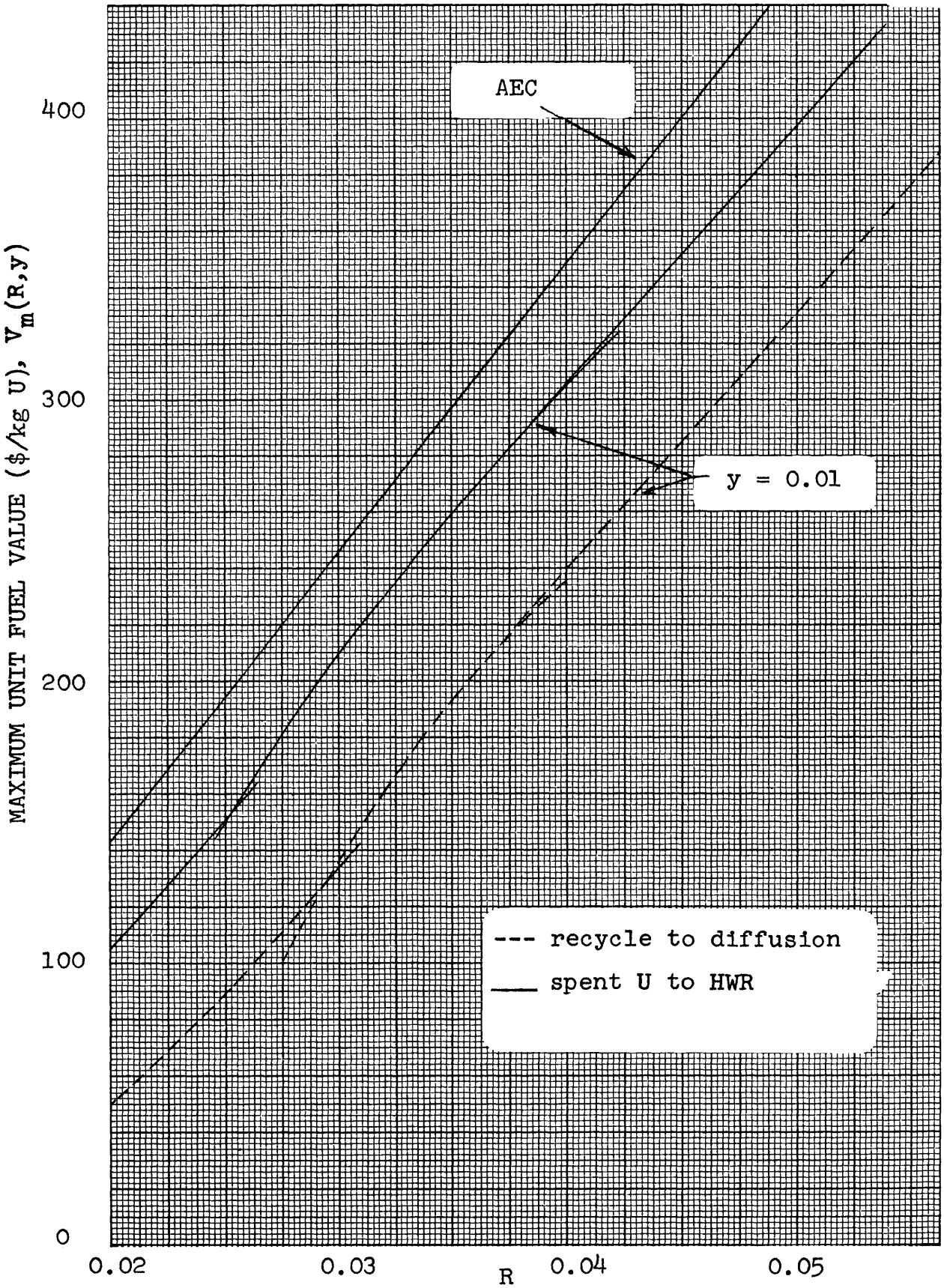


FIGURE II.7 Comparison of the Maximum Unit Fuel Value for PWR with Spent Fuel Going to HWR and with Spent U Recycled through Diffusion Plant
$C_{U_3O_8}$ = \$8/lb C_N = \$0/g

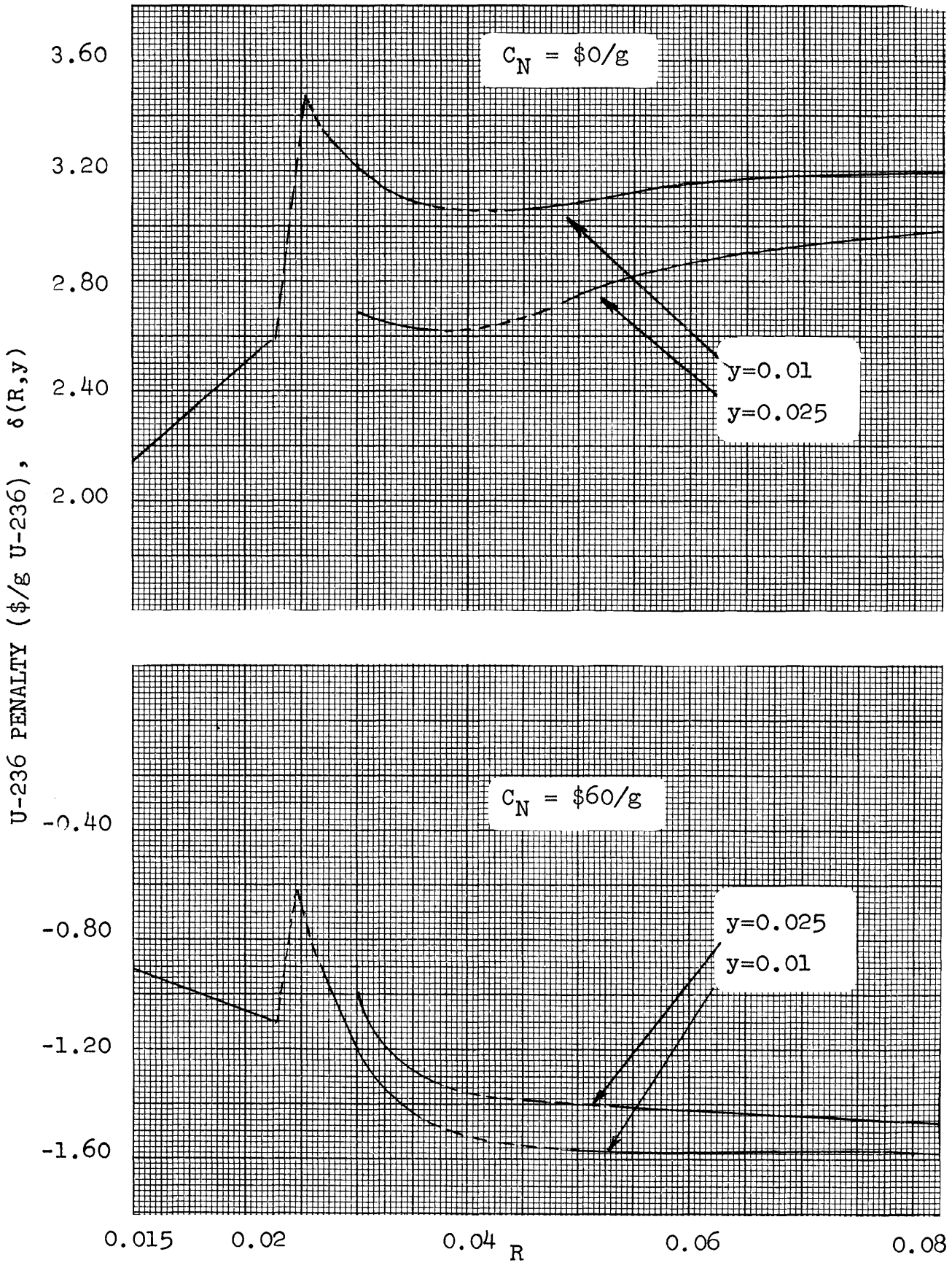


FIGURE II.8 The Effect of R and y on the U-236 Penalty - PWR with Spent U Fed to HWR $C_{U_3O_8}$ = \$8/lb

C. Summarized Comparison

Table II.2 compares representative values of the U-236 penalties and the neptunium indifference values for the cases dealt with in this report with those treated in the earlier report (1):

TABLE II.2

Summary of U-236 Penalties and Neptunium Indifference Values†

Reactor	HWR	PWR		
		Recycle (1)		
Disposition of Spent U	Discarded	To HWR	Thru Diff. Plant	To Fabrication
Optimum U-235/U-238 Weight Ratio R*	0.0136	0.032	0.031	0.55
U-236 Penalty, δ, \$/g U-236				
C_N = \$0/g	1.2	3.0	10	26
60	-1.0	-1.3	- 1	-11
Neptunium Indifference Value, \$/g	32	44	55	43

† At y = 0.01, R = R*, $C_{U_3O_8}$ = \$8/lb.

Each neptunium indifference value represents the sale price for neptunium present in spent fuel leaving the indicated type of reactor at which the total fuel cycle cost would be unaffected by the presence of 1 w/o U-236 in the feed to the system. Yhr corresponding market price for neptunium equals this indifference value plus the incremental cost of recovering neptunium from spent fuel.

III. HEAVY WATER REACTOR

The heavy water reactor used as a reference to examine the effect of U-236 on power plant economics is a 1,000 Mwe HWOCR; it is very similar, though not identical, to the conceptual design jointly proposed by Combustion Engineering, Inc. and Atomics International Division of North American Aviation, Inc. for the U. S. Atomic Energy Commission.(4) Some of the reference HWOCR characteristics are listed in Appendix B; Reference (4) was relied on heavily in the preparation of this appendix. One of the salient features of this reactor is its high fuel utilization: its ability to produce a large amount of energy per unit of fissile isotope expended. After initial startup, continuous, bidirectional on-line refueling takes place utilizing uranium carbide fuel of low enrichment. The utilization of separate fuel channels for continuous on-line refueling limits the excess reactivity that is ever present in the reactor and thus minimizes the number of neutrons which are lost to parasitic capture in control poison. In addition parasitic absorption in the moderator of a large HWOCR is limited to one or two percent of the neutrons. It is thus obvious that neutron economy plays an important role in the design considerations of an HWOCR.

The following is a more detailed description of the reference HWOCR.

A. HWOCR Description

1. General

The reactor vessel, calandria, is cylindrical with a vertical orientation and is constructed of austenitic stainless steel. The overall dimensions of the calandria are an outside diameter of 25.0 feet and an inside height of 20.0 feet. The radial wall thickness is 1.0 inches which gives an inside diameter of 24.83 feet. The inside dimensions of the calandria provides a 12 inch reflector in both axial and radial directions outside the active core. The heavy water moderator is contained in the cylindrical calandria.

Through tubes of Zircaloy-2 are rolled into the upper and lower end tube sheets on a 10.5 inch square lattice arrangement. A process tube of SAP-895 passes through each of the 492 calandria tubes and contains the five fuel element assemblies in tandem and the organic coolant. The fuel is hyperstoichiometric uranium carbide clad in SAP and the coolant is a terphenyl mixture.

The core utilizes bidirectional refueling with the reactor at power; the fuel movement is in opposite directions in adjacent process tubes. Likewise the

coolant flow is bidirectional with the flow in the same direction as the fuel movement. The coolant makes only one pass through the core before flowing to the heat exchangers.

2. Fuel Element

Each fuel element consists of 37 SAP-clad fuel rods. The outside diameter (excluding fins) of the thirty-one larger rods is 0.521 in. while the other six have an outside diameter of 0.324 in.; the two different sized rods help achieve a circular configuration within the fuel elements. The cladding is free standing under the external pressure of the coolant and is 0.020 in. thick.

The outside diameter of the large UC slugs is 0.476 in. and the small slugs is 0.277 in. This leaves a radial fuel-clad gap of 0.0025 in. for the large rods and 0.0035 in. for the small rods; this gap is filled with helium.

Each fuel element is 43.2 in. long. Five of these are stacked to fill one process channel with 43.2 in. long spacer shields located above and below the fuel. The actual fuel length in the outer row of large rods is 41.13 in. while the actual fuel length of the inner rows of rods and the smaller rods is 41.75 in.

The fuel is hyperstoichiometric uranium carbide, nominally 5% by weight carbon, cast into slugs

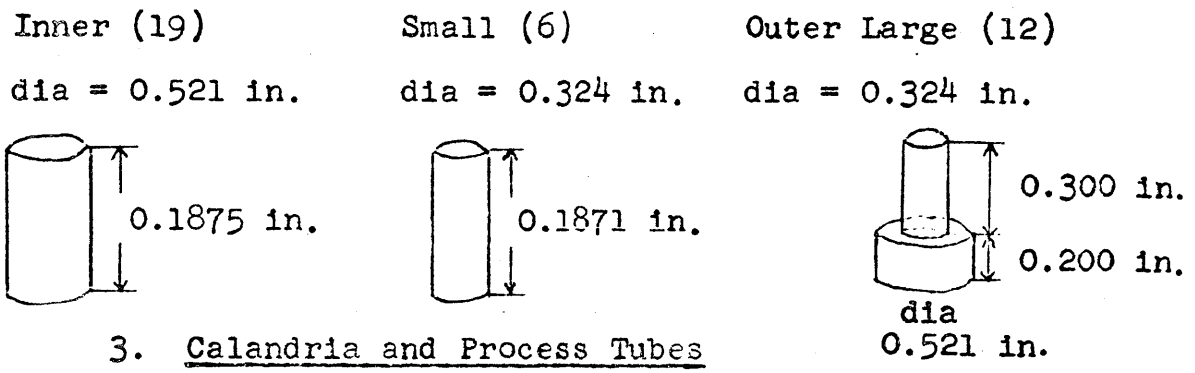
approximately three in. long. One end of the pellet is dished to provide a uniform bearing-surface on the pellet interface. The x-ray density of UC is approximately 13.6 gm/cm^3 but when packing density and gas expansion space is considered the density is about 13.0 gm/cm^3. The packing density excluding gas gaps is 13.34 gm/cm^3.

The large rods have 12 fins (0.080 in. high x 0.030 in. wide) equally spaced on the periphery of the tube and spiraling at 90^o/ft. The smaller rods have six axially straight fins (0.060 in. high x 0.030 in. wide) and two taller fins (0.128 in. high x 0.040 in. wide) which do not spiral.

The fuel rods are restrained at each end by Zircaloy-4 end plates attached to the twelve rods on the periphery of the bundle. (Figure II-6 of Volume II of Reference (2)) The details of the construction are contained in Reference (2). The Zr-4 end plates are 0.1875 in. thick and 4.260 in. in diameter. For the purposes of making volume calculations, it has been assumed that 2/3 of the total end plate volume is solid material and 1/3 is open space in the form of orificing.

Short end caps are used on the six small rods and all inner floating rods, with longer end caps for the 12 outer rods. For the purpose of volume calculations,

it is assumed that the end caps have the shape as follows



3. Calandria and Process Tubes

The process tube, which is made of SAP-895, has an inside diameter of 4.32 in. and is 0.116 in. thick. The calandria tube, which is made of Zircaloy-2, has an inside diameter of 5.094 in. and is 0.052 in. thick. This leaves a gap annulus of 0.271 in. which is filled with CO_2 gas. These tubes extend the full length of the active core.

4. Fuel

The fuel is hyperstoichiometric UC which is compatible with its cladding, SAP, up to about 950°F which is significantly higher than the fuel-clad interface temperature.

Each fuel assembly contains 50 kilograms of uranium which leads to a total core loading of 123 metric tons of uranium.

Reference (7) indicated the effective fuel temperature at full power was 1,000°F. This value will be used throughout these calculations.

Reference (4) gives the cross sectional area of fuel in an assembly to be 5.85 in.

The fuel density including gas space is 13.0 g/cc and excluding the gas space is 13.34 g/cc hot.

5. Organic Coolant

The organic coolant is a mixture of terphenyl isomers marketed commercially as SANTOWAX-OM. The physical properties of irradiated SANTOWAX-OM have not been determined, but are expected to be very close to those of SANTOWAX-OMP which is used at the Piqua Nuclear Power Facility. (4) To obtain a balance between physical and heat transfer properties and the coolant decomposition rate, an equilibrium concentration of 10% high boilers is used.

The reactor inlet temperature of the organic coolant is 595°F and the reactor outlet temperature is 750°F. The average coolant temperature for calculation purposes will be 672.5°F. The total coolant flow is 110×10^6 lb/hr.

The density of the terphenyl with 10% high boiler content at 672.5°F is 0.837 grams per cm^3.

For the purpose of calculating the molecular weight of the terphenyl with 10% high boilers, the molecular weight of terphenyl with no high boilers is taken to be 230.31. The molecular weight range of the composition

of high boilers is given in Table III.1.(4)

TABLE III.1

Composition of High Boilers

Molecular Weight Range	Content (%)
226 - 268	6
269 - 344	6
345 - 420	10
421 - 496	73
497 - 572	1
573 - 648	<1
>648	3

The pressure at the reactor inlet header is 284 psia and there is a reactor pressure drop (header to header) of 184 psi.

6. Moderator

The moderator is D_2O which is maintained at a purity of 99.75% D_2O. The moderator is circulated through the reactor core; the calandria inlet temperature is 140°F and the outlet temperature is 200°F. The average temperature within the calandria is 190°F and this will be used for all calculations. (4)

The calandria contains 588,000 lbs of D_2O.

7. Power

The total fission power including moderator and shield heat loss is 3093 Mwth. The net plant efficiency is 34.7% and the plant produces a net plant power output of 1073 Mwe. The maximum heat release is 26.7 kw/ft.

B. CELL and MOVE Code Application

The CELL and MOVE computer codes were utilized in order to predict the behavior of the HWOCR system; both of these codes have been developed at MIT. The CELL code, which calculates the homogenized unit cell properties, nuclide concentrations and criticality parameters, as a function of flux time, is described in detail in Reference (8). The MOVE code, which is described in detail in Reference (9), utilizes the flux-time properties calculated by CELL, which can be transferred to MOVE by magnetic tape or punched card, and core geometry input data to obtain fuel, flux, and

power density behavior during fuel burnup for a specific fuel management scheme - in this case continuous bidirectional fuel management. In the continuous bidirectional fuel management scheme, fuel moves at a constant axial velocity along a channel from one end of the reactor where it is charged to the opposite end where it is discharged. Fuel moves in opposite directions in adjacent channels, and the charging rate is adjusted so as to maintain criticality without the use of control poison.(9)

The input data for CELL and MOVE and the methods used to obtain it are described in Appendix C.

In order to verify that CELL and MOVE were adequately predicting the equilibrium behavior of the reference HWOCR, calculations were made with CELL and MOVE using the same fuel enrichment as had been used by AI-CE.(4) In addition, Oak Ridge National Laboratory had made some calculations using the AI-CE reference design.(5) Hence comparisons of the results from CELL and MOVE with those obtained by AI-CE and ORNL would give an indication of how well CELLMOVE was functioning; this is particularly important because AI-CE and ORNL used computer codes which are more intricate and time consuming and which would be expected to predict results close to reality. A comparison of the results is shown in Table III.2.

TABLE III.2

Comparison of Equilibrium Condition Depletion

Calculation for the HWOCR

	Feed Enrichment (w/o U-235)	Discharge Enrichment (w/o U-235)	Burnup (MWD/T)	Discharge Fissile Pu (gm/kg U)	Ave. Excess Reactivity
CELLMOVE	1.16	0.128	17,043	3.22	0.0
AI-CE[4]	1.16	0.205	15,000	3.16	
ORNL (One-Dimensional Code)[5]	1.159	0.197	16,009	3.34	0.008
ORNL (Normalized)*[5]	1.16		16,801		0.0

*From tables listed in Reference (5), it was determined that 0.001 excess reactivity corresponded to a loss of about 96 MWD/T while 0.001% change in fuel enrichment corresponded to a 24 MWD/T change in burnup.

It can be seen from Table III.2 that the reactivity lifetime predicted by CELLMOVE is higher than that predicted by AI-CE. This can probably be attributed to the fact that a continuous fuel management technique is being employed by CELLMOVE which is only an approximation of reality. Since the fuel management scheme is actually discontinuous (five fuel assemblies per channel), there will be some neutron loss to control mechanisms which would decrease reactivity lifetime. It can be seen that when the average excess reactivity is removed from ORNL one dimensional calculation, there is agreement on reactivity lifetime to about 1.5%.

The difference in discharge enrichment is primarily a result of the differences in reactivity lifetime. Even with the differences in reactivity lifetime, the difference involved is less than 8% over the range of U-235 depletion.

There appears to be very good agreement with AI-CE and ORNL on the amount of fissile plutonium present at discharge.

It was thus concluded that CELLMOVE was adequately predicting the reactivity lifetime and discharge fuel composition for the reference HWOCR. With this confidence in the CELLMOVE calculations, a number of runs were made over a range of R and y values. R is the

the weight ratio of U-235 to U-238 uranium feed; the range of R values were from 0.008 to 0.020. y is the weight fraction of U-236 in the uranium feed; the range of y values were from 0.0 to 0.030. One additional case was considered, that being the case of natural uranium fed to the reactor; for this case it was found that the reactor would operate but would achieve only about 2,500 MWD/T burnup which would make the operation of the HWOCR on natural uranium very uneconomical. The results obtained from the CELLMOVE runs are tabulated in Table III.3. In all cases of reasonably high burnup, the discharge enrichment is so low that the spent uranium has no economic value, i.e. for burnups greater than 7,000 MWD/MT, the discharge enrichments are less than 0.3 w/o.

In Figure III.1, burnup has been plotted as a function of R (with y as a parameter). As would be expected, the slope of the curves indicates that the amount of increase in reactivity lifetime per unit increase in R decreases with increasing R. It can also be seen that the effect of adding U-236 is to decrease the reactivity lifetime as would be expected. Careful examination reveals that this effect decreases with increasing amounts of U-236.

TABLE III.3

Fuel Cycle Performance of HWOCR

R	y	Burnup (MWD/MT)	Flowrate (kg/day)	Fissile Plutonium Discharged (g/initial kgU)	Np-237 Discharged (g/initial kgU)
0.00716	0.0	2,513	984.69	1.60	0.007
0.008	0.0	7,151	346.02	2.70	0.029
0.010	0.0	12,951	191.06	3.13	0.066
0.012	0.0	17,672	140.02	3.25	0.102
0.014	0.0	21,496	115.11	3.31	0.134
0.016	0.0	25,072	98.69	3.33	0.167
0.020	0.0	30,272	81.74	3.31	0.224
0.008	0.005	5,433	455.44	2.41	0.149
0.010	0.005	11,578	213.72	3.06	0.281
0.012	0.005	16,208	152.67	3.23	0.345
0.014	0.005	20,147	122.82	3.30	0.423
0.016	0.005	23,575	104.96	3.33	0.472
0.020	0.005	28,728	86.13	3.41	0.545
0.008	0.010	4,161	594.66	2.11	0.201
0.010	0.010	10,451	236.76	2.99	0.432
0.012	0.010	15,075	164.14	3.20	0.560
0.014	0.010	18,965	130.47	3.29	0.649
0.016	0.010	22,491	110.02	3.33	0.720
0.020	0.010	27,668	89.43	3.41	0.814

TABLE III,3

(Continued)

R	y	Burnup (MWD/MT)	Flowrate (kg/day	Fissile Plutonium Discharged (g/initial kgU)	Np-237 Discharged (g/initial kgU)
0.010	0.020	8,650	286.06	2.82	0.617
0.012	0.020	13,247	186.79	3.13	0.840
0.014	0.020	17,172	144.10	3.25	0.992
0.016	0.020	20,679	119.66	3.31	1.107
0.020	0.020	25,609	96.62	3.39	1.244
0.010	0.030	7,221	342.67	2.63	0.712
0.012	0.030	11,828	209.20	3.04	1.029
0.014	0.030	15,503	159.61	3.20	1.231
0.016	0.030	19,131	129.34	3.28	1.397
0.020	0.030	24,207	102.22	3.35	1.583

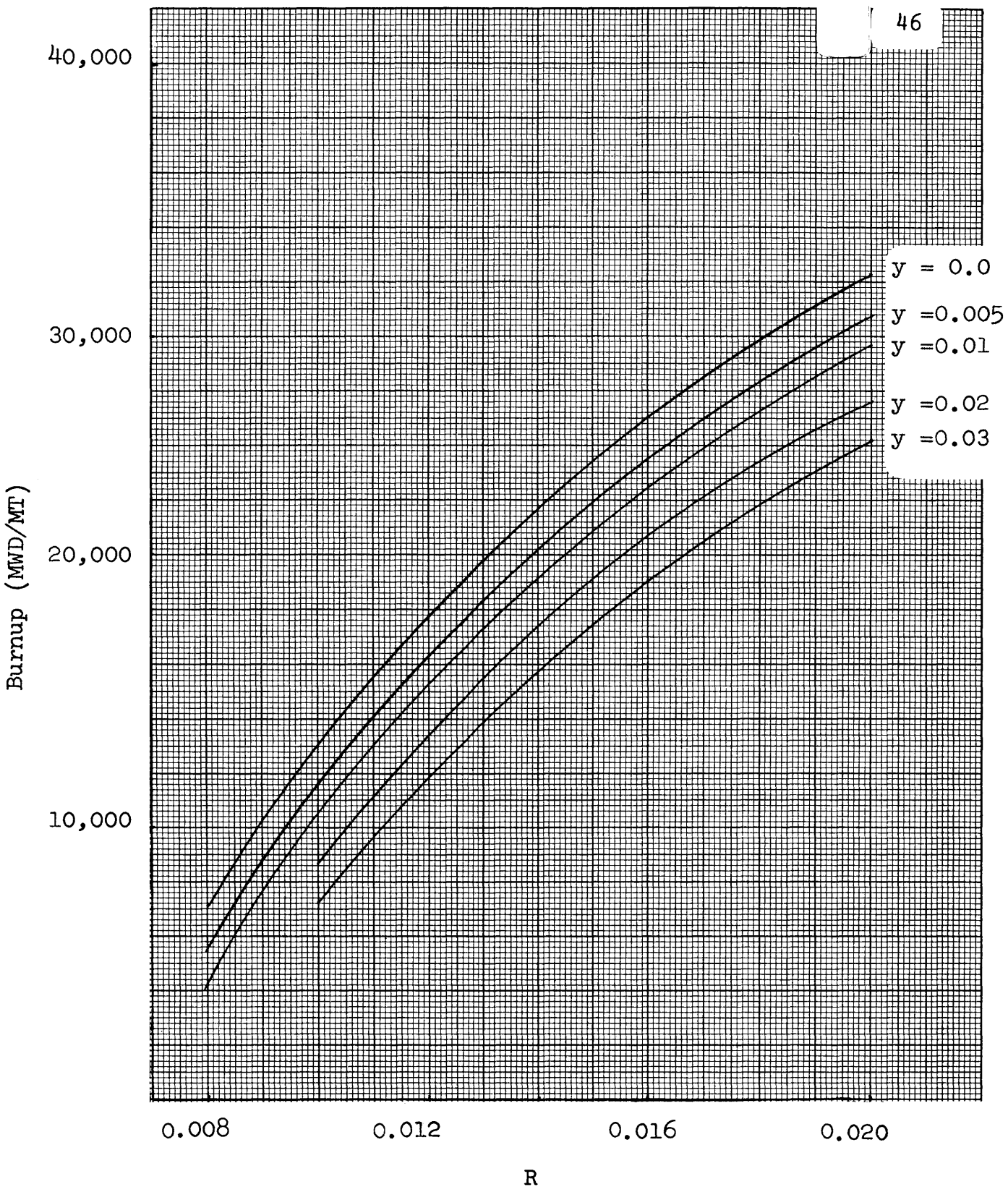


FIGURE III.1 The Effect of R and y on Burnup in HWOCR

IV. CALCULATION OF OPTIMUM FUEL CYCLE COSTS IN HWOCR

A. Economic Parameters

In a study such as this, it is necessary to assume a set of economic parameters in order to calculate fuel cycle costs. It is also necessary to project these costs into the future in order to adequately represent equilibrium fuel cycle costs at a time when the reactor in question, if built, would be operating. These projections are very difficult to make primarily because of the strong economic dependence on the size of the industry which is to be served.

It was assumed that the economic parameters should represent conditions in the late 1970's and should be based on reasonably large scale processing geared to an expanding HWOCR industry. In addition, an attempt was made not to be either overly optimistic or pessimistic in regard to future costs; in situations where projections were not clear a degree of conservatism was exercised in the choice of economic parameters. The parameters used in this study have been given in Table I.1.

The price of U_3O_8 will be an economic variable in this study. The \$8/lb represents current AEC pricing while the value of \$6/lb is closer to the present world market price. It has been forcasted that the world market price of uranium is likely to rise in the

future and for this reason the third value of $10/lb of U_3O_8 was chosen for study.

The estimated future value of Np-237 is far less certain and depends upon the development of radioisotopic space power systems and the use of Pu-238 as a radioisotopic fuel. Since the effect of U-236 on the value of uranium feed is expected to be very dependent on the price of Np-237 and since it is essentially impossible to forecast the future value of Np-237, it was decided that the price of Np-237 should be an economic variable. Np-237 price is therefore varied from a minimum of $0/g to a maximum of $100/g; intermediate values also used are $20/g and $60/g. It is difficult at this time to foresee circumstances where the value of Np-237 would be greater than $100/gm, thus it was felt that the range of Np-237 prices would be representative for some time into the future.

AI-CE projected fabrication and shipping costs for the HWOCR to be $40/kgU for the initial core and about $36/kgU for replacement fuel.[4] ORNL portrayed fabrication costs that ranged from $31/kgU to $34/kgU[5] while Kasten indicated that more recent estimates were for costs less than $30/kgU.[10] Since ORNL values were judged to be optimistic, a more conservative value of $40/kgU was chosen for this study.

In regards to reprocessing costs, AI-CE predicted costs of $18/kgU[4] while ORNL portrayed costs ranging from $19/kgU to $24/kgU.[5] Kasten[10] indicated that forecasted reprocessing costs for UO_2 were about $25/kgU - $30/kgU, and since there was no reason to assume that UC reprocessing would be cheaper than UO_2 reprocessing, a reprocessing cost of $25/kgU was decided on for this work.

ORNL[5] used $2.74/kgU for spent fuel shipping and Kasten[10] indicated that this number was obtained after a very detailed analysis. Hence the rounded value of $3/kgU was used for this study.

Kasten[10] and AI-CE[4] were in agreement that $3/kgU was a good value for fuel storage in lieu of

recovery and this value was used when it proved uneconomical to reprocess spent fuel. Fuel losses of 1% during fabrication and 1% during reprocessing have been widely used in fuel cycle analysis and were chosen for use here. The pre-irradiation holdup time of 0.2 years and post-irradiation holdup time of 0.67 years that were used by AI-CE seemed reasonable and were therefore taken directly from Reference (4). The pre-irradiation holdup time may appear to be low but this is associated with continuous on-line refueling used by the HWOCR.

The annual charge on fuel inventory was taken to be 10% per annum and the load factor was assumed to be 0.80.

B. Minimum Fuel Cycle Costs

It is now possible to proceed with the determination of the minimum fuel cycle cost when the uranium feed does not contain any U-236. By determining steady state fuel cycle costs for some discrete feed enrichments and then utilizing interpolation methods, the minimum fuel cycle cost as well as the optimum R of the feed can be determined.

Since fuel cycle costs are highly dependent on economic parameters which are projections, it was felt

that a reasonably simple fuel cycle model could be utilized for the calculation of fuel cycle costs. In addition this study is not concerned with the absolute value of the fuel cycle costs for the HWOCR, but rather the effect of U-236 on the fuel cycle cost.

The following is a description of some of the individual cost items which when combined will give the fuel cycle cost. All costs listed are in dollars per initial kilogram of uranium. The cost of the uranium which is purchased and enriched by the AEC is $\frac{C_{AEC}(R)}{(1-L_F)}$ and the cost of fabrication is C_F. The interest on the inventory during fabrication will be taken to be based on the combined value of the uranium and the cost of fabrication and is equal to $it_F \left(\frac{C_{AEC}(R)}{1-L_F} + C_F\right)$. Interest during irradiation will be charged on the mean value of the reactor inventory during irradiation; this is equivalent to an interest charge equal to $\frac{it_R}{2} \left(\frac{C_{AEC}(R)}{1-L_F} + C_F\right)$ for the first half of reactor residence time, t_R, and equal to $\frac{it_R}{2} (- C_{POST})$ during the second half where C_{POST} is the cost of reprocessing, C_A, plus the cost of shipping, C_{SH}, less the plutonium and neptunium credit. If C_{POST} is greater than storage in lieu of recovery, C_{STOR}, the Pu and Np are not recovered and $C_{POST} = C_{STOR}$. The discharge enrichment of the uranium

from the HWOCR is so low that it had essentially no value. The credit for the neptunium is equal to $(1 - L_{RP})Q_N C_N$ where Q_N is the number of grams of Np-237 discharged from the reactor per initial kilogram of uranium and the credit for the fissile plutonium is equal to $(1 - L_{RP})Q_K C_K$ where Q_K is the number of grams of fissile plutonium discharged from the reactor per initial kilogram of uranium. The interest charge on the plutonium and neptunium inventory during reprocessing is it_{RP} $(- C_{POST})$.

The net fuel cycle cost in dollars per initial kilogram of uranium is given by the following expression:

$$C_{\$}(R) = \left[\frac{C_{AEC}(R)}{(1-L_F)} + C_F\right](1 + it_F + \frac{it_R}{2}) + C_{POST}(1 - \frac{it_R}{2} - it_{RP}) \qquad (IV.1)$$

where $C_{POST} = C_A + C_{SH} - (1-L_{RP})(Q_K C_K + Q_N C_N)$ (IV.2)

or $= C_{STOR}$ (whichever is smaller)

The net fuel cycle cost in mills per kilowatt-hour is then given by

$$C_E(R) = \frac{C_{\$}(R) \times 1000}{24 \times \eta \times B} \qquad (IV.3)$$

where **B** is the burnup in megawatt-days per metric ton and η is the thermal efficiency of the plant.

The above equation was then utilized to determine fuel cycle costs as a function of R for twelve cases; these twelve cases are the result of using three different U_3O_8 prices in conjunction with four different Np-237 prices. The results are tabulated in Table IV.2 .

By examining the fuel cycle costs in Table IV.2, it can be seen that the R giving the minimum fuel cycle cost is reasonably close to .014 for all cases. In order to determine the minimum fuel cycle costs, a parabolic interpolation was performed using the three lowest fuel cycle cost points; in all cases this corresponded to the points R = .012, R = .014 and R = .016. The interpolation yielded the minimum fuel cycle cost as well as the optimum weight ratio, R*, associated with it. Interpolation minimum values are listed in Table IV.3.

As would be expected, the minimum fuel cycle cost decreases with decreasing U_3O_8 price and with increasing Np-237 price.

C. Shifts in the Optimum R

In evaluating fuel cycle economics, the unit total direct costs - net material costs plus fabrication, reprocessing and conversion costs - tend to decrease with increasing burnup over a wide burnup range. However the unit carrying charges tend to increase with increasing

TABLE IV.2

Fuel Cycle Costs, $C_E(R)$, as a Function of Prices of Natural Uranium and Neptunium (HWOCR)

Case	Natural Uranium Price, $C_{U_3O_8}$ ($/lb)	Neptunium Price, C_N ($/g)	Fuel Cycle Cost (mills/kwh)					
			R = Natural Uranium	0.008	0.012	0.014	0.016	0.020
1	10	0	3.553	1.357	0.8605	0.8606	0.8775	0.9669
2	10	20	3.553	1.349	0.8493	0.8489	0.8654	0.9541
3	10	60	3.553	1.332	0.8270	0.8255	0.8413	0.9285
4	10	100	3.553	1.315	0.8047	0.8021	0.8173	0.9030
5	8	0	3.288	1.282	0.7938	0.7895	0.8022	0.8805
6	8	20	3.288	1.273	0.7827	0.7778	0.7901	0.8677
7	8	60	3.288	1.256	0.7604	0.7544	0.7660	0.8422
8	8	100	3.288	1.239	0.7381	0.7310	0.7420	0.8166
9	6	0	3.024	1.193	0.7258	0.7165	0.7244	0.7908
10	6	20	3.024	1.193	0.7146	0.7048	0.7123	0.7780
11	6	60	3.024	1.182	0.6923	0.6814	0.6882	0.7524
12	6	100	3.024	1.164	0.6700	0.6580	0.6641	0.7269

TABLE IV.3

Minimum Fuel Cycle Cost at Different Prices for Natural Uranium and Neptunium (HWOCR)

Case	Natural U_3O_8 Price, $C_{U_3O_8}$ ($/lb)	Np-237 Price, C_N ($/g Np-237)	Optimum Weight Ratio, R*	Minimum Fuel Cycle Cost, C_E^* (mills/kwh)
1	10	0	0.01299	0.8584
2	10	20	0.01305	0.8470
3	10	60	0.01317	0.8240
4	10	100	0.01329	0.8010
5	8	0	0.01351	0.7890
6	8	20	0.01357	0.7774
7	8	60	0.01368	0.7542
8	8	100	0.01378	0.7309
9	6	0	0.01408	0.7165
10	6	20	0.01413	0.7048
11	6	60	0.01423	0.6813
12	6	100	0.01433	0.6578

burnup due to the fact that higher burnups requires higher fissile inventories. The sum of the total direct costs and the carrying charges is the total fuel cycle costs, and the result of the interplay between the two is a minimum fuel cycle cost occurring at some optimum R.

Examination of Table IV.3 shows that the optimum R decreases with increasing U_3O_8 price. The higher the U_3O_8 price, the greater the investment in fuel and therefore the carrying charges will be greater. Since the fabricating, reprocessing and conversion costs remain constant, the effect of the higher U_3O_8 price is to decrease the optimum R which will decrease the proportion of fuel cycle costs which are carrying charges in relation to the non-optimum higher R case.

In regards to changes in Np-237 price, the optimum R increases with increasing price. In this case the added credit at fuel discharge due to the Np-237 (the Np-237 content increases with burnup) more than overcomes the carrying charge increase due to the added discharge inventory and the tendency is to increase optimum burnup with increasing Np-237 price.

V. MODES OF OPERATION

One basic mode of operation and two modifications of this mode of operation, any of which can be employed by a heavy water moderated reactor operator, have been investigated in this study. It will be shown later that each of these modes has economic advantage under certain circumstances.

What is referred to as the base case mode of operation is illustrated in Figure V.1; it is a simple once-through cycle with no credit received for spent uranium, due to its low discharge enrichment, and with plutonium and neptunium recovered only when economically justified. The other two modes of operation are modification of this base case and require the base case fuel value results for fuel valuation.

Throughout it is assumed that the reactor operator has the opportunity to purchase fuel of composition R,y and that it is his desire to determine how much he can afford to pay for it. In the flowsheet illustrated in Figure V.1 uranium of composition R,y is purchased in the form of UO_3 and is fed directly to the fabricator at the flowrate F. Fabrication losses are at the rate $L_F F_R$, and feed of composition R_R, y_R is fed to the reactor at the flowrate F_R. The reactor generates P Mw(e) power and discharges the fuel which is eventually fed to

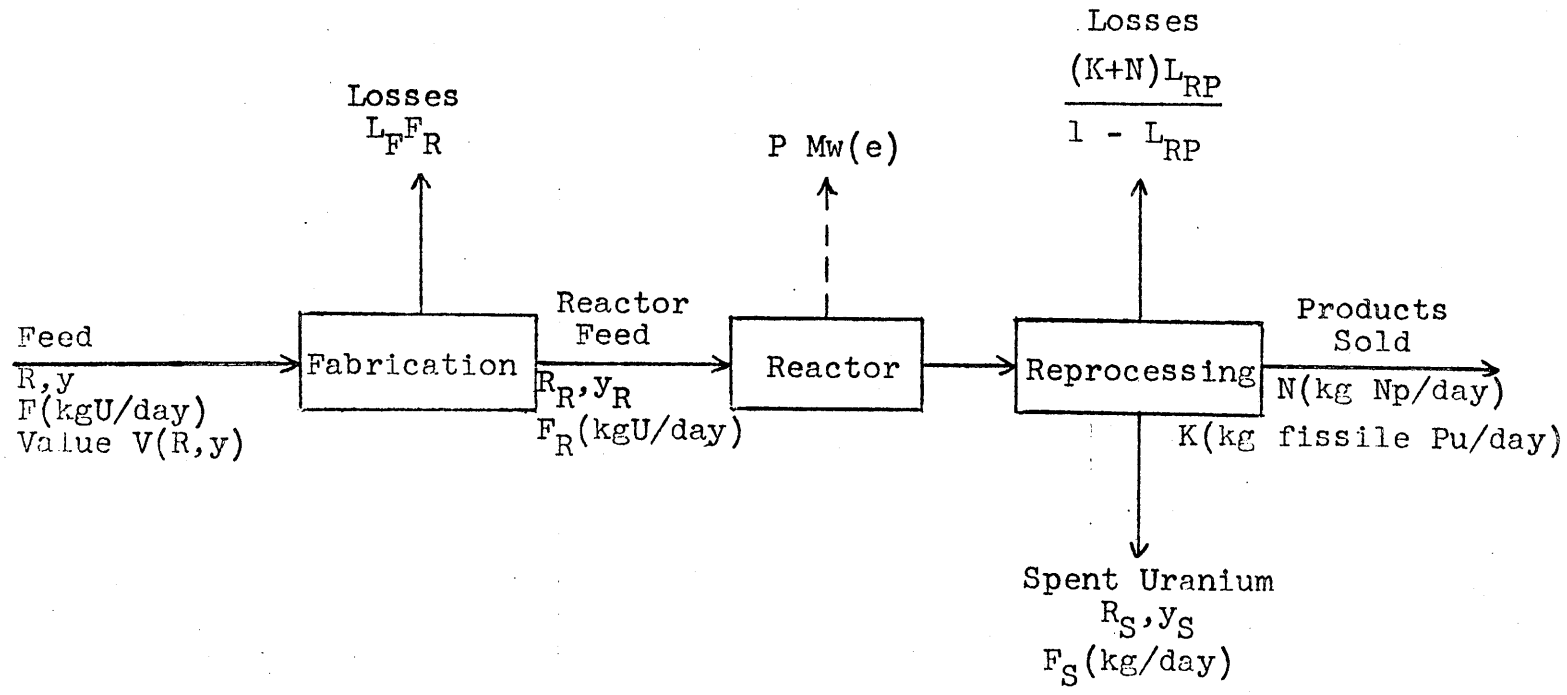


FIGURE V.1 Base Case Flow Scheme

the reprocessing plant. Spent uranium of composition R_S, y_S, which no longer is of any value, is discharged from the reprocessing plant at the rate, F_S. Losses of fissile plutonium and neptunium in reprocessing are at the rate $\frac{(K+N)L_{RP}}{1 - L_{RP}}$, and fissile plutonium and neptunium are sold at rates K and N respectively. If K and N are so low that reprocessing is not economic, a storage charge is made in lieu of reprocessing and no credit is given for plutonium and neptunium. The value of the uranium feed is determined by assuming that the overall fuel cycle costs for the scheme illustrated in Figure V.1 is the same as the minimum fuel cycle costs, C_E^*, obtained in Section IV for the same $C_{U_3O_8}$ and C_N.

A modification of the basic mode of operation is applicable when the uranium for which a value is to be determined has an R which is less than R*. This mode of operation pre-enriches the uranium by gaseous diffusion and allows valuation of uranium of very low R (the lower limit being R_T) as well as uranium with an R sufficiently high so that it could be evaluated using the basic case mode of operation. The flowsheet for the pre-enrichment by gaseous diffusion mode of operation is given in Figure V.2. An economic assumption which was made for simplification is that the fabricator's cost of converting UO_3 to UC is the same as the

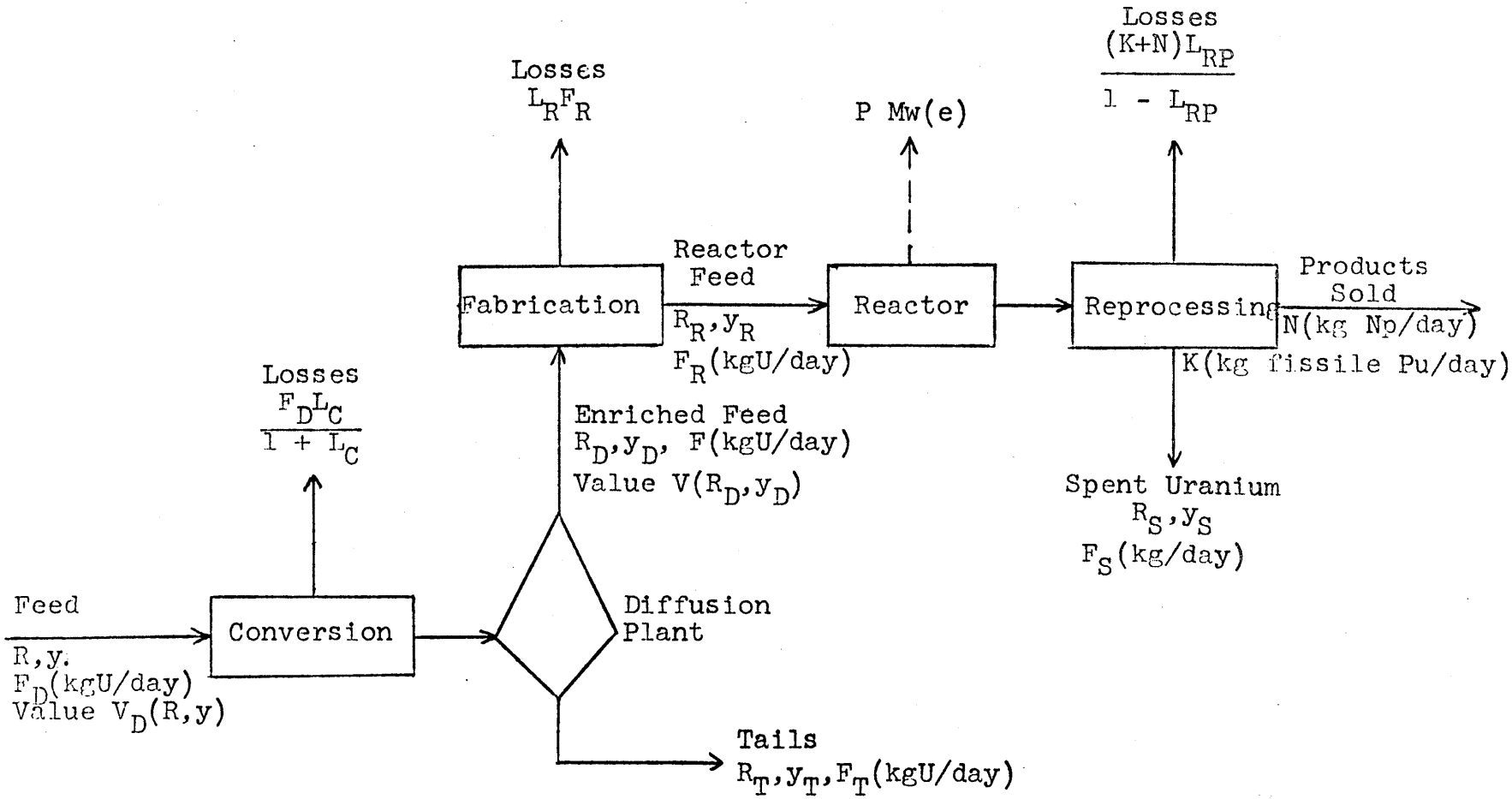


FIGURE V.2 Modified Case with Pre-Enrichment by Gaseous Diffusion

cost of converting UF_6 to UC. There is no evidence to indicate that this is not a satisfactory assumption. As is seen in the flowsheet, the uranium of composition R,y which is to be purchased is converted from UO_3 to UF_6 and then fed at the flowrate $F_D(\frac{1}{1+L_C})$ into a gaseous diffusion plant for enrichment to the composition R_D, y_D. The composition of the diffusion plant tails is R_T, y_T and the tails flowrate is F_T. The diffusion plant heads are supplied to the fabricator as UF_6 at the flowrate F; after the material reaches the fabricator the flowsheet is identical to that shown in Figure V.1.

Another modification of the basic mode of operation is applicable when the uranium for which a value is to be determined has an R greater than R*. This mode of operation blends the uranium with natural uranium as UF_6 priced on the AEC scale.[1] The flowsheet for this modified case is given in Figure V.3. As can be seen, uranium of composition R,y with flowrate F_B is blended with natural uranium with flowrate F_{NAT} so that uranium with a composition R_B, y_B is fed to the fabricator at a flowrate F. After the blending has been accomplished,

1. This is not meant to imply that the natural uranium used for blending must be fed to fabrication as UF_6. What is assumed is that if natural uranium concentrates are converted directly to UC, the cost of conversion is the same as the sum of conversion from U_3O_8 to UF_6 plus conversion from UF_6 to UC. The simplification which results more than justifies the assumption because fabrication costs include converting either UF_6 or UO_3 to UC.

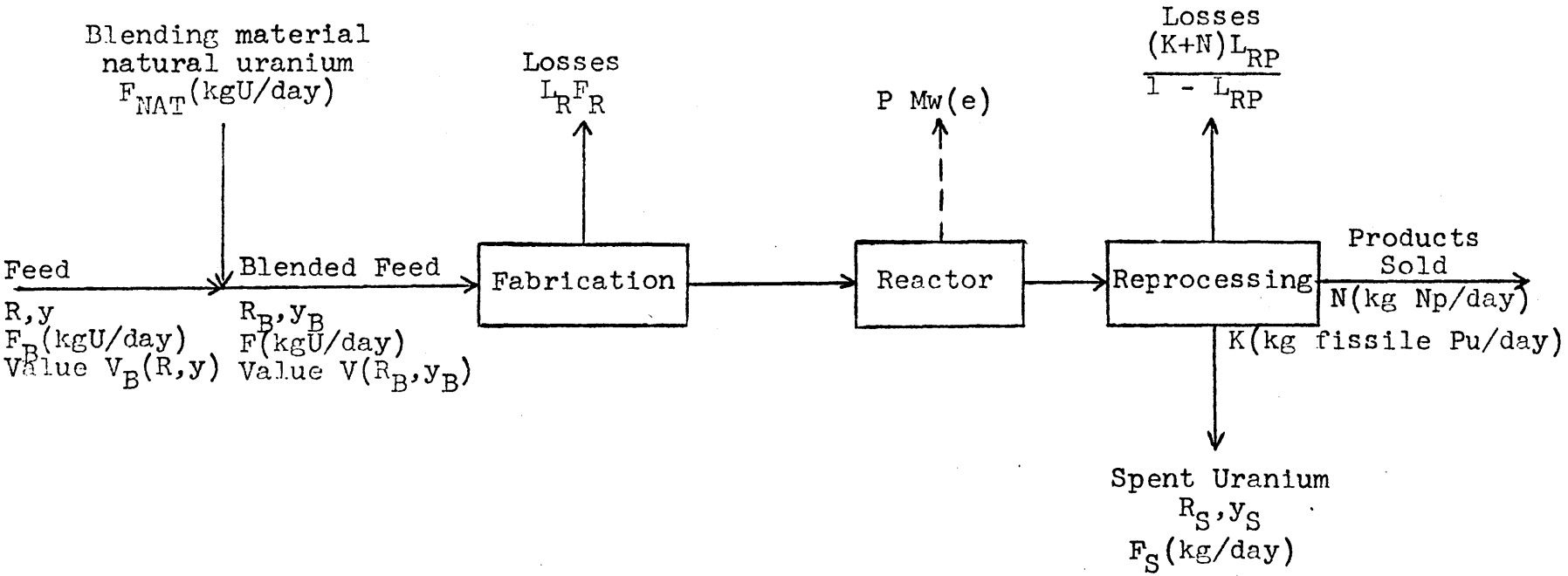


FIGURE V.3 Modified Case with Blending with Natural Uranium

the flowsheet is identical to that shown in Figure V.1.

Another method of operation which has not been examined in this study is available to the reactor operator under some circumstances. When the material to be purchased has an R less than R*, it might be advantageous to blend this uranium with other uranium having an R greater than R*; this latter uranium could be obtained either from the AEC or from another reactor operator. The advantage of this blending method would be highly dependent on the composition and quantities of uranium available and the purchasing arrangements which could be obtained.

VI. VALUATIONAL PRINCIPLES FOR URANIUM IN HWR

A. Base Case

When the reactor is fed with uranium containing y weight fraction U-236 and R weight ratio of U-235 to U-238, the value of this feed V(R,y) is to be found from the condition that the fuel cycle cost C_E in mills/kwh is to be equal to the minimum fuel cycle cost C_E^* when the same reactor is fed with uranium free from U-236 of optimum enrichment R^* priced on the AEC price scale.

The net fuel cycle cost C_E in mills/kwh for the base case shown in Figure V.1 is

$$C_E = \frac{1000\left[\left(C_F + \frac{V(R,y)}{1-L_F}\right)\left(1 + it_F + \frac{it_R}{2}\right) + C_{POST}\left(1 - \frac{it_R}{2} - it_{RP}\right)\right]}{24\eta B} \qquad \text{(VI.1)}$$

The derivation is similar to Eqs. (IV.1) and (IV.3), except that the desired fuel value V(R,y) has been used in place of the value on the AEC scale $C_{AEC}(R)$. The result of setting C_E in Eq. (VI.1) equal to C_E^* and solving for V(R,y) is:

$$V(R,y) = (1-L_F)\left[\frac{0.024\eta B C_E^* - C_{POST}\left(1 - \frac{it_R}{2} - t_{RP}\right)}{1 + it_F + \frac{it_R}{2}} - C_F\right] \qquad \text{(VI.2)}$$

For each of the twelve cases, using Equation (VI.2), a complete set of fuel values can be obtained for the range of R and y of interest.

B. Pre-enrichment by Diffusion

When feed uranium of composition R,y is pre-enriched by gaseous diffusion prior to fabrication, as illustrated in Figure V.2, there is one optimum composition of diffusion plant product (R_D,y_D) fed to fabrication which leads to a maximum value of uranium feed $V_D(R,y)$. $V_D(R,y)$ is related to the unit value of diffusion plant product $V(R_D,y_D)$, which is known from the base case uranium valuation, by a cost balance on the conversion operation and the diffusion plant:

$$FV(R_D,y_D) = (F_D V_D(R,y))(1+it_C) + F_D C_{CT} + \Delta_D C_\Delta + it_E FV(R_D,y_D) \qquad \text{(VI.3)}$$

Here F is time-average flow rate of diffusion plant product fed to fabrication, in kg/day,

F_D is time-average flow rate of uranium fed to conversion, in kg/day,

t_C is time interval between purchase of UO_3 and conversion to UF_6, in years,

t_E is time interval between delivery of UF_6 to AEC for toll enrichment and receipt of diffusion plant product, in years,

C_{CT} is the cost of conversion of UO_3 to UF_6, in \$/kg U,

C_Δ is the cost of separative work, in \$/kg U,

and

Δ_D is the amount of separative work expended in pre-enriching uranium.

The result of solving Equation (VI.3) for the cost of uranium feed $V_D(R,y)$ is

$$V_D(R,y) = \frac{(1-it_E)FV(R_D,y_D) - F_D C_{CT} - \Delta_D C_\Delta}{F_D(1+it_C)} \qquad \text{(VI.4)}$$

As all the quantities on the right are known (Appendix D gives additional diffusion plant equations used to determine some of these quantities), $V_D(R,y)$ can be determined. With a given R and y, R_D is varied, y_D is determined from the known characteristics of the diffusion plant operated as a matched-R cascade[11], and $V_D(R,y)$ at that R_D is evaluated. The R_D which results in a maximum value of $V_D(R,y)$ is the optimum, and this value of $V_D(R,y)$ is the desired result.

C. Blending with Natural Uranium

When uranium feed of composition R,y is to be blended with natural uranium prior to fabrication, as illustrated in Figure V.2, there is an optimum fraction, ε, of natural uranium to be used in blending which leads to a maximum value of uranium feed $V_B(R,y)$. $V_B(R,y)$ is related to the unit value of uranium after blending $V(R_B,y_B)$ by the cost balance equation

$$V_B(R,y) = \frac{V(R_B,y_B) - \varepsilon C_{NAT}}{1 - \varepsilon} \qquad \text{(VI.5)}$$

where C_{NAT} is the cost of natural uranium on the AEC price scale. As the uranium after blending is fed to fabrication, its unit value $V(R_B,y_B)$ is known from the base case analysis. Also, y_B is related to y by the U-236 material balance equation

$$y_B = (1-\varepsilon)y \qquad \text{(VI.6)}$$

and R_B is related to R by the following equation which is derived with the aid of the U-235 material balance relation:

$$R_B = \frac{\frac{R(1-y)(1-\varepsilon)}{1+R} + \frac{R_{NAT}\,\varepsilon}{1+R_{NAT}}}{1 - \frac{R(1-y)(1-\varepsilon)}{1+R} - \frac{R_{NAT}\,\varepsilon}{1+P_{NAT}} - (1-\varepsilon)y} \qquad \text{(VI.7)}$$

The procedure to find the maximum value of uranium feed of composition R,y then is to select a blending fraction ε, solve for y_B from Equation (VI.6), solve for R_B from Equation (VI.7) find $V(R_B,y_B)$ from the base case result, and solve for $V_B(R,y)$ from Equation (VI.5). The value of ε which leads to the maximum value of $V_B(R,y)$ is the optimum, and this maximum value is the desired final value of $V_B(R,y)$.

VII. RESULTS FOR BASE MODE OF OPERATION OF HWR

A reexamination of the results obtained from CELL and MOVE codes, Table III.3 shows that for a fixed U-235 to U-238 weight ratio in the feed, as the amount of U-236 increases (assuming U-236 replaces U-235 plus U-238), the burnup as well as the amount of fissile plutonium in the spent fuel decreases. However with increasing y the amount of Np-237 in the spent fuel increases. From this one would generalize that for C_N = \$0 or some low value, the value of feed would decrease with increasing U-236 in the feed for a given $C_{U_3O_8}$. This is precisely the result which is obtained. The base case fuel values, V(R,y), calculated using Equation VI.2 are tabulated in Table VII.1 for the twelve cases which have been considered. In addition, graphical representation of V(R,y) is shown in Figures VII.1, VII.2, VII.3 and VII.4 for cases 5, 7, 9 and 11. The line marked "AEC" in these figures is the price of UF_6 as a function of R for a unit cost of \$30/kgU for separative work and the indicated cost of U_3O_8, on the AEC scale.

As has been shown, these base case fuel values, V(R,y), are essential in the determination of fuel values using modified modes of operation.

It is of interest to discuss the general features of the base case curves. First it should be noted that the y = 0 unit fuel value curve is tangent to the curve

TABLE VII.1

Calculated Unit Fuel Values, V(R,y), ($/kg U) **Base Case HWR**

Base Case 1: $C_{U_3O_8}$ = \$10/lb, C_N = \$0/g

	R = 0.008	0.010	0.012	0.014	0.016	0.020
y = 0.00	8.72	47.50	75.15	96.02	114.50	139.72
0.005	-4.83	38.91	66.88	88.84	106.94	133.01
0.01	-14.08	31.58	60.28	82.42	101.33	127.85
0.02		19.18	49.23	72.37	91.73	117.59
0.03		8.60	40.23	62.70	83.27	110.30

Base Case 2: $C_{U_3O_8}$ = \$10/lb, C_N = \$20/g

	R = 0.008	0.010	0.012	0.014	0.016	0.020
y = 0.00	8.56	47.40	75.12	96.06	114.61	140.00
0.005	-2.83	42.20	70.46	92.90	111.11	137.27
0.01	-12.91	37.36	67.14	89.77	108.91	135.53
0.02		28.23	60.61	84.91	104.87	131.06
0.03		19.49	54.85	79.14	100.83	128.49

Base Case 3: $C_{U_3O_8}$ = \$10/lb, C_N = \$60/g

	R = 0.008	0.010	0.012	0.014	0.016	0.020
y = 0.00	8.24	47.16	75.03	96.12	114.80	140.52
0.005	1.15	48.77	77.59	101.00	119.42	145.75
0.01	-6.87	48.90	80.83	104.46	124.05	150.87
0.02		46.31	83.35	109.98	131.11	157.97
0.03		41.27	84.07	112.01	135.90	164.86

Base Case 4: $C_{U_3O_8}$ = \$10/lb, C_N = \$100/g

	R = 0.008	0.010	0.012	0.014	0.016	0.020
y = 0.00	7.92	46.93	74.94	96.18	114.99	141.04
0.005	5.13	55.33	84.73	109.10	127.73	154.23
0.01	-0.82	60.45	94.52	119.14	139.18	166.20
0.02		64.40	106.10	135.04	157.35	184.88
0.03		63.05	113.29	144.88	170.98	201.22

Base Case 5: $C_{U_3O_8}$ = \$8/lb, C_N = \$0/g

	R = 0.008	0.010	0.012	0.014	0.016	0.020
y = 0.00	2.84	38.62	64.15	83.42	100.49	123.79
0.005	-8.69	30.69	56.51	76.79	93.51	117.58
0.01	-16.35	23.92	50.42	70.86	88.33	112.82
0.02		12.49	40.22	61.58	79.46	103.34
0.03		2.73	31.91	52.65	71.65	96.61

TABLE VII.1

(Continued)

Base Case 6: $C_{U_3O_8}$ = \$8/lb, C_N = \$20/g

	R = 0.008	0.010	0.012	0.014	0.016	0.020
y = 0.00	2.67	38.50	64.10	83.43	100.57	124.03
0.005	-7.66	33.97	60.07	80.83	97.65	121.80
0.01	-16.73	29.69	57.25	78.19	95.88	120.46
0.02		21.52	51.58	74.10	92.57	116.78
0.03		13.61	46.51	69.08	89.18	114.77

Base Case 7: $C_{U_3O_8}$ = \$8/lb, C_N = \$60/g

	R = 0.008	0.010	0.012	0.014	0.016	0.020
y = 0.00	2.33	38.25	63.98	83.46	100.72	124.50
0.005	-3.69	40.51	67.18	88.90	105.92	130.24
0.01	-10.83	41.22	70.92	92.85	110.98	135.76
0.02		39.59	74.30	99.14	118.78	143.65
0.03		35.38	75.71	101.92	124.22	151.10

Base Case 8: $C_{U_3O_8}$ = \$8/lb, C_N = \$100/g

	R = 0.008	0.010	0.012	0.014	0.016	0.020
y = 0.00	1.99	37.98	63.85	83.48	100.86	124.96
0.005	0.28	47.05	74.28	96.96	114.19	138.66
0.01	-4.79	52.74	84.58	107.49	126.07	151.04
0.02		57.66	97.02	124.16	144.98	170.50
0.03		57.14	104.91	134.76	159.26	187.41

Base Case 9: $C_{U_3O_8}$ = \$6/lb, C_N = \$0/g

	R = 0.008	0.010	0.012	0.014	0.016	0.020
y = 0.00	-2.54	29.33	52.64	70.24	85.84	107.13
0.005	-11.76	22.09	45.66	64.19	79.46	101.45
0.01	-18.72	15.91	40.10	58.77	74.72	97.10
0.02		5.48	30.78	50.29	66.62	88.44
0.03		-2.17	23.20	42.14	59.49	82.29

Base Case 10: $C_{U_3O_8}$ = \$6/lb, C_N = \$20/g

	R = 0.008	0.010	0.012	0.014	0.016	0.020
y = 0.00	-3.18	29.19	52.57	70.24	85.90	107.35
0.005	-12.26	25.35	49.21	68.21	83.58	105.65
0.01	-19.11	21.67	46.92	66.08	82.26	104.72
0.02		14.50	42.14	62.80	79.72	101.86
0.03		7.46	37.80	58.55	77.00	100.43

TABLE VII.1

(Continued)

Base Case 11: $C_{U_3O_8}$ = \$6/lb, C_N = \$60/g

	R = 0.008	0.010	0.012	0.014	0.016	0.020
y = 0.00	-3.85	28.91	52.42	70.23	86.00	107.76
0.005	-8.76	31.87	56.28	76.24	91.81	114.03
0.01	-14.98	33.18	60.56	80.70	97.32	119.96
0.02		32.56	64.83	87.80	105.89	128.68
0.03		29.21	66.97	91.36	112.01	136.71

Base Case 12: $C_{U_3O_8}$ = \$6/lb, C_N = \$100/g

	R = 0.008	0.010	0.012	0.014	0.016	0.020
y = 0.00	-4.20	28.63	52.27	70.21	86.10	108.18
0.005	-4.80	38.39	63.36	84.27	100.04	122.41
0.01	-8.95	44.68	74.19	95.32	112.37	135.20
0.02		50.61	87.52	112.80	132.06	155.50
0.03		50.96	96.15	124.18	147.02	172.99

which represents the AEC price scale at R = R* and lies below the AEC price scale at all other values of R; this is the result of assuming the overall fuel cycle costs when evaluating feed must be equal to the minimum fuel cycle costs, C_E^*, using fuel having no U-236 purchased as UF_6 on the AEC price scale and assuming that the cost of converting UO_3 to UC is equal to the cost of converting UF_6 to UC.

By examining Figure VII.1 where neptunium has no value, it is seen that increasing the U-236 content decreases the fuel value as expected; it can also be seen that the effect of a given quantity of U-236 decreases as the total quantity of U-236 increases. By contrast, Figure VII.2, where neptunium is valued at \$60/g, shows that for $R > 0.011$, the effect of increasing y is to increase the value of the fuel. At $R < 0.009$, the presence of U-236 acts as poison but the residence time of the fuel in the reactor is not long enough for sufficient quantities of Np-237 to build up to econonomically overcome the poisoning effect of the U-236 and as a result the presence of U-236 decreases the value of the fuel. It can generally be stated that for a given R and a given high Np-237 price, continually increasing y will not continually increase the fuel value, for at some y value the poisoning effect causing

decreased reactivity lifetime will override the increase in Np-237 credit.

Another interesting aspect which can be investigated is the exact effect of changing C_N on the fuel values, V(R,y); this can be accomplished by calculating the change in fuel value as a function of Np-237 price for the three intervals \$0/g to \$20/g, \$20/g to \$60/g, and \$60/g to \$100/g. The results of such an analysis of the change in uranium value per \$/g change in Np-237 price for R = 0.014 and y = 0.00, 0.005, 0.01, 0.02, and 0.03 are shown in Table VII.2 for the three values of $C_{U_3O_8}$. From the information in Table VII.2 and from similar analysis for other R values, it can be seen that the fuel value, V(R,y), is linearly dependent on C_N.

The effect of changing $C_{U_3O_8}$ holding C_N constant can be visually observed by comparing Figures VII.1 and VII.3 where C_N = \$0/g and by comparing Figures VII.2 and VII.4 where C_N = \$60/g. In particular, decreasing the natural U_3O_8 price does not appreciably change the shape of any of the curves but tends to shift both the fuel value curves and the AEC price scale to lower values.

There is also a slight tendency toward non-linear variation of uranium value with $C_{U_3O_8}$ as is seen in Table VII.2. This is confirmed in Table VII.3 which

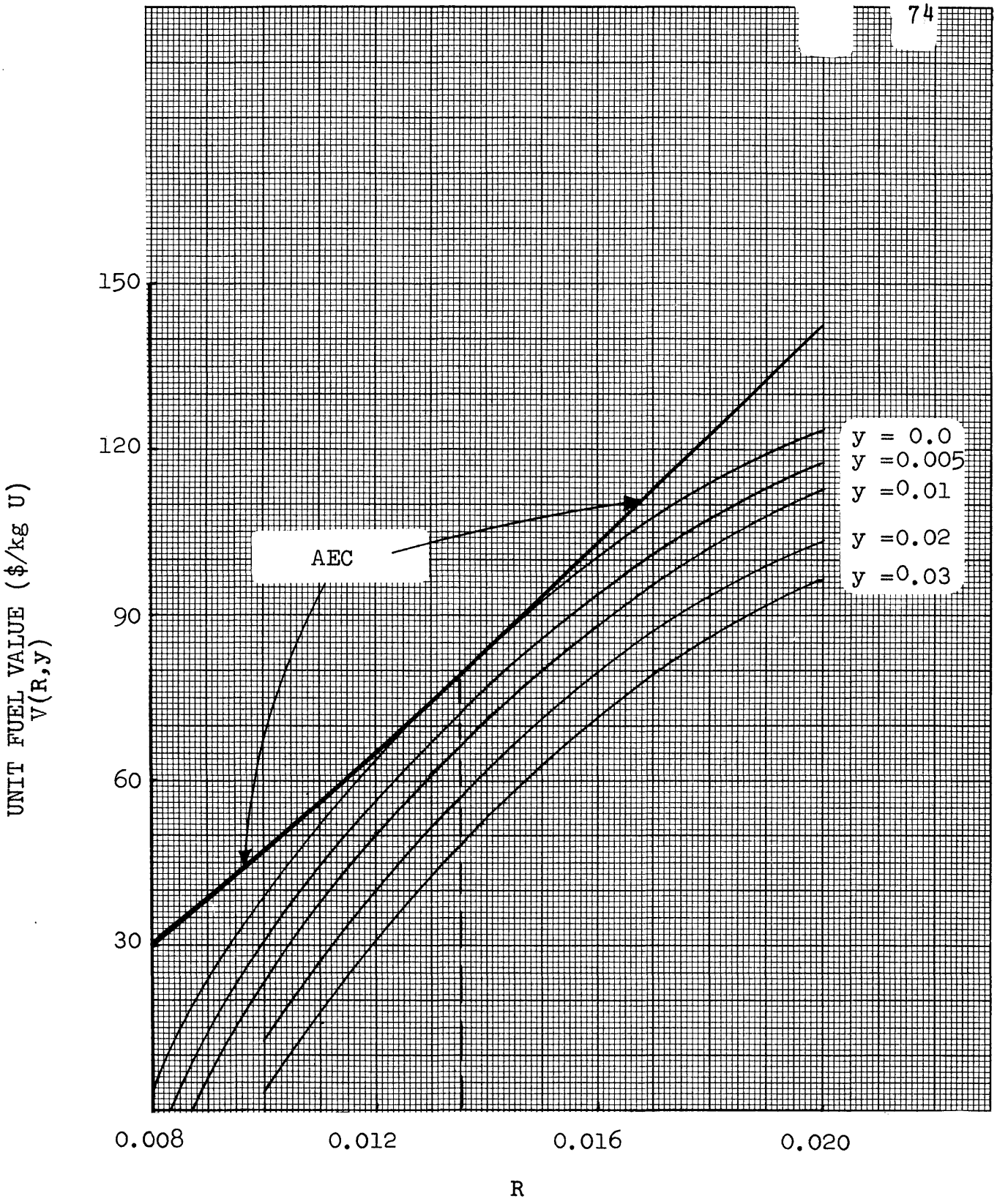


FIGURE VII.1 The Effect of R and y on Base Case Unit Fuel Value - $C_{U_3O_8}$ = \$8/lb, C_N = \$0/g. HWR.

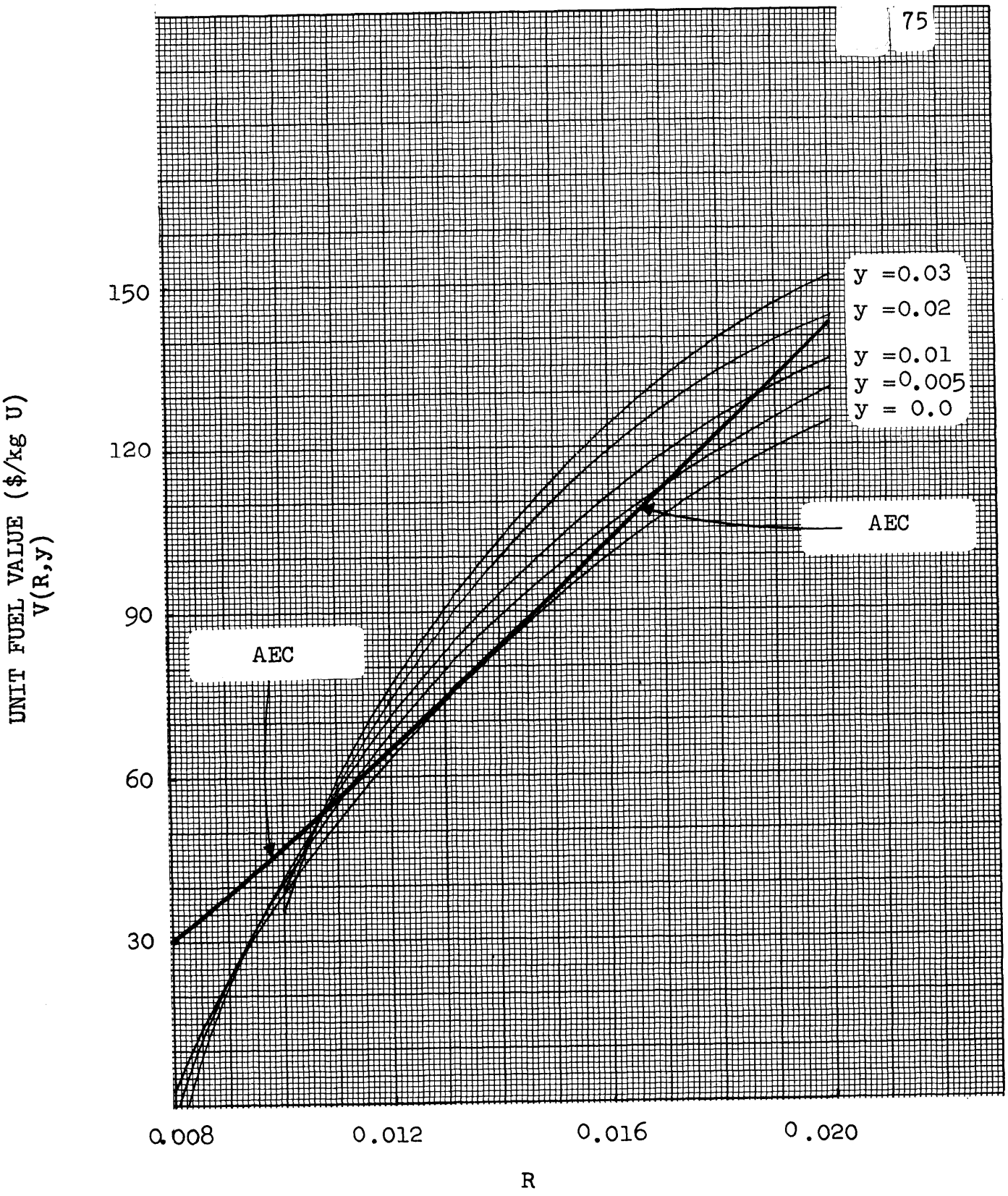


FIGURE VII.2 The Effect of R and y on Base Case Unit Fuel Value - $C_{U_3O_8}$ = \$8/lb, C_N = \$60/g . HWR.

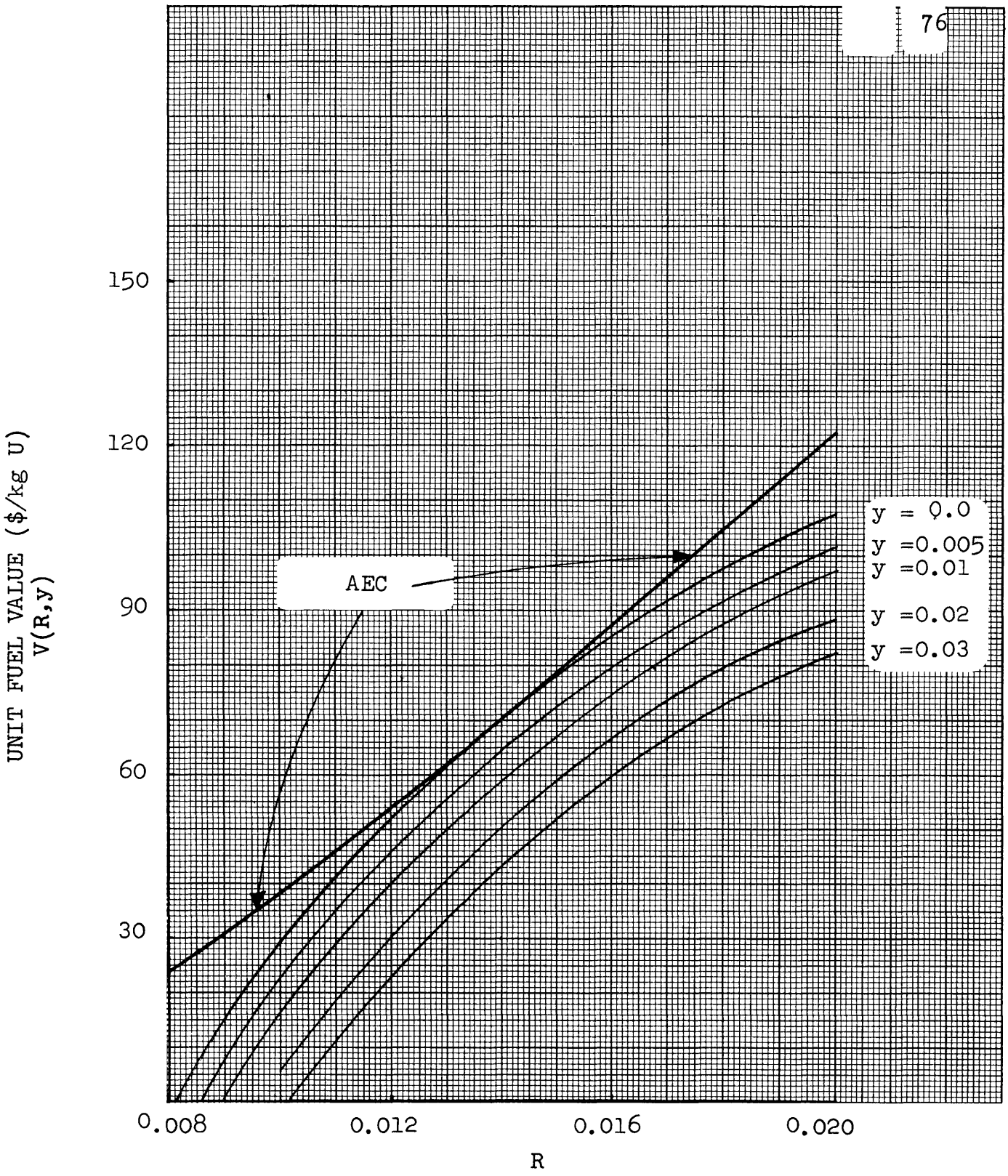


FIGURE VII.3 The Effect of R and y on Base Case Unit Fuel Value - $C_{U_3O_8}$ = \$6/lb, C_N = \$0/g . HWR.

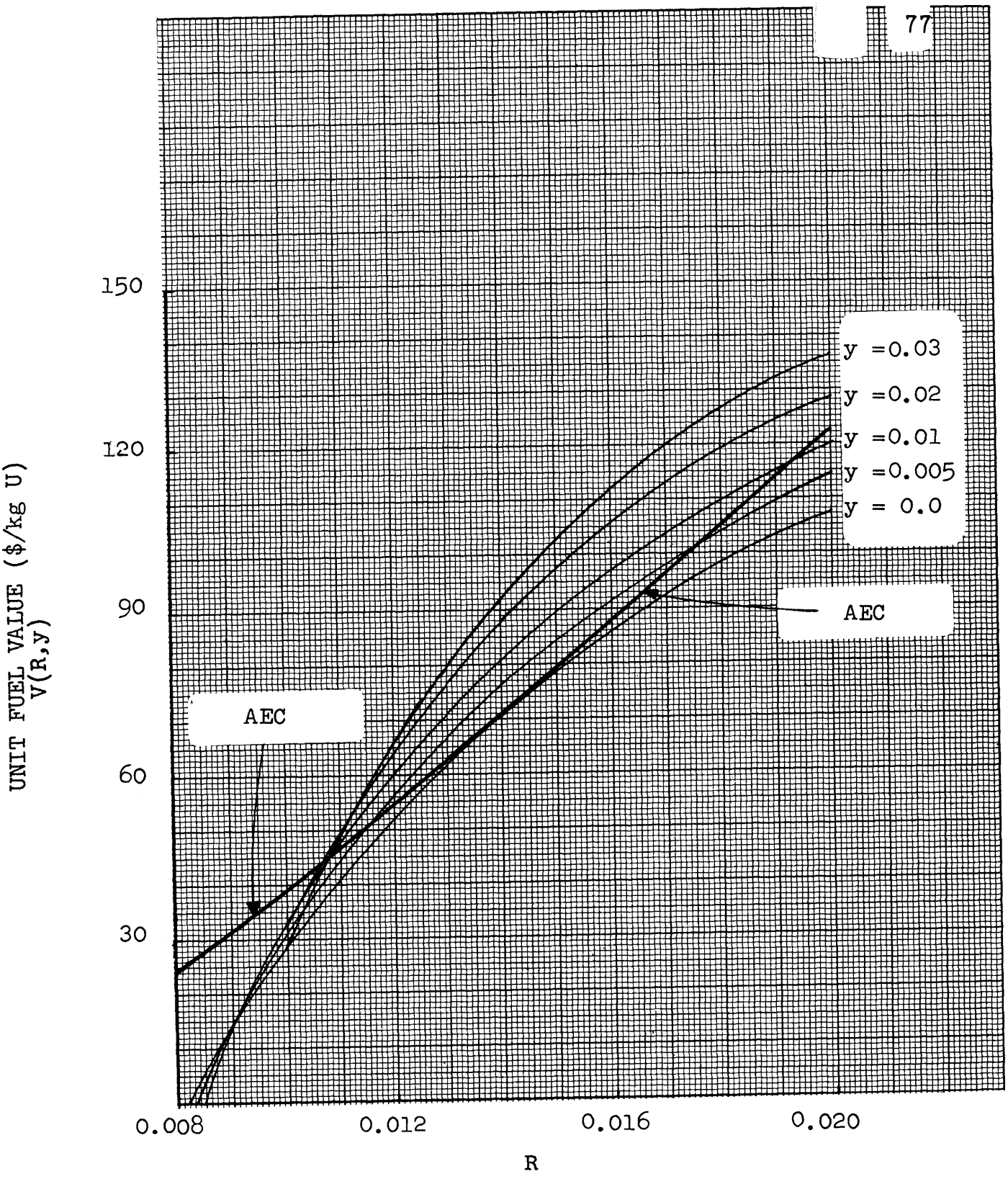


FIGURE VII.4 The Effect of R and y on Base Case Unit Fuel Value - $C_{U_3O_8}$ = \$6/lb, C_N = \$60/g. HWR.

TABLE VII.2

Change of Fuel Value with Price of Neptunium. HWR.

$\frac{\Delta V(R,y)}{\Delta C_N}$($/kg U/g Np-237) at R = 0.014

	ΔC_N = \$20-\$0	ΔC_N = \$60-\$20	ΔC_N = \$100-\$60
$C_{U_3O_8}$ = \$10/lb			
y = 0.00	0.002	0.002	0.002
0.005	0.203	0.203	0.203
0.01	0.368	0.367	0.367
0.02	0.627	0.627	0.627
0.03	0.822	0.822	0.822
$C_{U_3O_8}$ = \$8/lb			
y = 0.00	0.001	0.001	0.001
0.005	0.202	0.202	0.202
0.01	0.367	0.367	0.366
0.02	0.626	0.626	0.626
0.03	0.822	0.821	0.821
$C_{U_3O_8}$ = \$6/lb			
y = 0.00	0	0	0
0.005	0.201	0.201	0.201
0.01	0.366	0.366	0.366
0.02	0.626	0.625	0.625
0.03	0.821	0.820	0.821

shows the change in uranium value per unit change in U_3O_8 price with C_N = \$0/g and R = 0.014. Since the linearity of V(R,y) with C_N has already been shown, this shows the general non-linearity of V(R,y) with $C_{U_3O_8}$ for any C_N. The reason for this non-linearity is that as $C_{U_3O_8}$ changes so do the optimum tails in the diffusion plant and hence one would not expect linearity.

TABLE VII.3

Change of Fuel Value with Price of U_3O_8. HWR.

	$\frac{\Delta V(R,y)}{\Delta C_{U_3O_8}}(\frac{\$/kg\ U}{\$/lb\ U_3O_8})$ at R = 0.014	
	$\Delta C_{U_3O_8}$ = \$8 - \$6	$\Delta C_{U_3O_8}$ = \$10 - \$8
y = 0.00	13.21	12.62
0.005	12.60	12.05
0.01	12.09	11.56
0.02	11.29	10.79
0.03	10.51	10.05

VIII. MAXIMUM FUEL VALUES IN HWR

As previously mentioned, the modes of operation illustrated in Figures V.2 and V.3 are a modification of what has been called the base case. In the modified case with pre-enrichment by gaseous diffusion, the range of R values that was examined was from R = 0.004 to R = 0.014 and the range of y values was from y = 0 to y = 0.02; the fuel values, $V_D(R,y)$, calculated using Equation VI.4, and other pertinent parameters that were calculated for eight cases are listed in Appendix E. For y = 0.02 and low R, there are no results listed because y_D would have been greater than 0.03 and would have required extrapolation of the base case fuel values, V(R,y), for which y = 0.03 was the greatest amount of U-236 considered.

In the modified case with feed uranium blended with natural uranium, results were generally obtained for R = 0.016, 0.018, and 0.020 with y = 0, 0.005, 0.01 and 0.02. The fuel values, $V_B(R,y)$, calculated using Equation VI.5, the fraction of natural uranium used in blending, ε, and other pertinent parameters that were calculated for seven cases are listed in Appendix F.

The modified fuel values, $V_D(R,y)$ and $V_B(R,y)$, are of interest in relation to the base case fuel values, V(R,y).

For a given composition R,y, the most economically advantageous mode of operation is the one which results in the highest value for fuel as determined from V(R,y), $V_D(R,y)$, and $V_B(R,y)$. The highest of these values $V_m(R,y)$ is the maximum price the reactor operator could afford to pay for this composition without having his cost of generating electricity exceed the cost when his fuel consists of uranium with no U-236 present of optimum enrichment purchased on the AEC price scale. $V_m(R,y)$ thus is the maximum value of feed uranium of this composition to the HWOCR operator. We therefore define $V_m(R,y)$ as the greatest of V(R,y), $V_D(R,y)$, and $V_B(R,y)$. Using the results reported in Table VII.1, Appendix E, and Appendix F, a set of maximum fuel values, $V_m(R,y)$ for seven cases have been obtained; these maximum fuel values are reported in Table VIII.1. Maximum fuel value curves similiar to Figures VII.1 through VII.2 have been prepared for cases 5 and 7 in Figures VIII.1 and VIII.2. The primary difference is that the complete V(R,y) curves, $V_B(R,y)$ curves, and most of the $V_D(R,y)$ have been represented for y = 0 and y = .01 while only those parts of a given y curve which are of highest value actually correspond to the maximum fuel value. This portrayal was chosen so that the relationship between V(R,y) (non-dashed line), $V_D(R,y)$ (long-dashed line), and $V_B(R,y)$ (short-dashed line) could be visualized.

TABLE VIII.1

Maximum Unit Fuel Values in HWR, $V_m(R,y)$ ($/kg U)

R =	0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020
Case 1: $C_{U_3O_8}$ = $10/lb, C_N = $0/g								
y = 0.00	13.28	29.90	48.36	(75.15)	(96.02)	115.53	134.96	154.31
0.005	9.21	24.65	42.21	(66.88)	(88.84)	108.19	127.48	146.70
0.01	6.47	20.78	37.37	(60.28)	(82.42)	101.57	120.49	139.47
0.02				(49.23)	(72.37)	91.73	109.70	127.61
Case 3: $C_{U_3O_8}$ = $10/lb, C_N = $60/g								
y = 0.00	13.27	29.88	48.34	(75.03)	(96.12)	115.65	135.10	154.48
0.005	15.70	32.99	51.94	(77.59)	(101.00)	120.77	140.27	159.68
0.01	17.48	32.28	54.64	(80.83)	(104.46)	124.54	144.22	163.74
0.02			58.74	(83.35)	(109.98)	131.22	150.97	170.36
Case 5: $C_{U_3O_8}$ = $8/lb, C_N = $0/g								
y = 0.00	9.41	23.85	40.15	(64.15)	(83.42)	100.87	118.19	135.45
0.005	5.92	19.24	34.70	(56.51)	(76.79)	94.18	111.37	128.52
0.01	3.57	15.89	30.44	(50.42)	(70.86)	88.34	105.10	121.98
0.02				(40.22)	(61.58)	(79.46)	95.67	111.78
Case 7: $C_{U_3O_8}$ = $8/lb, C_N = $60/g								
y = 0.00	9.39	23.83	40.12	(63.98)	(83.46)	100.99	118.34	135.63
0.005	12.07	27.36	44.28	(67.18)	(88.90)	106.77	124.21	141.53
0.01	14.01	29.91	47.33	(70.92)	(92.85)	111.33	128.73	146.20
0.02			52.02	(74.30)	(99.14)	118.81	136.78	154.34
Case 8: $C_{U_3O_8}$ = $8/lb, C_N = $100/g								
y = 0.00	9.38	23.81	40.10	(63.85)	(83.48)	101.07	118.44	135.75
0.005	16.19	32.78	50.67	(74.28)	(96.96)	115.15	132.76	150.21
0.01	21.03	39.42	58.75	(84.58)	(107.49)	126.35	144.47	162.32
0.02			70.82	(97.02)	(124.16)	145.08	164.21	182.72
Case 9: $C_{U_3O_8}$ = $6/lb, C_N = $0/g								
y = 0.00	5.56	17.74	31.75	(52.64)	(70.24)	85.87	101.02	116.15
0.005	2.78	13.81	27.02	(45.65)	(64.19)	79.68	94.66	109.61
0.01	0.78	10.09	23.40	(40.10)	(58.77)	(74.72)	89.22	103.84
0.02				(30.78)	(50.29)	(66.62)	81.02	95.23
Case 11: $C_{U_3O_8}$ = $6/lb, C_N = $60/g								
y = 0.00	5.59	17.72	31.72	(52.42)	(70.23)	86.01	101.19	116.32
0.005	8.42	21.61	36.40	(56.28)	(76.24)	92.21	107.45	122.59
0.01	10.47	24.40	39.77	(60.56)	(80.70)	(97.32)	112.62	127.89
0.02			45.05	(64.83)	(87.80	(105.89)	121.99	137.68

Values in parentheses are from base case. Values at lower R are from pre-enrichment by gaseous diffusion. Values at higher R are from blending with natural uranium.

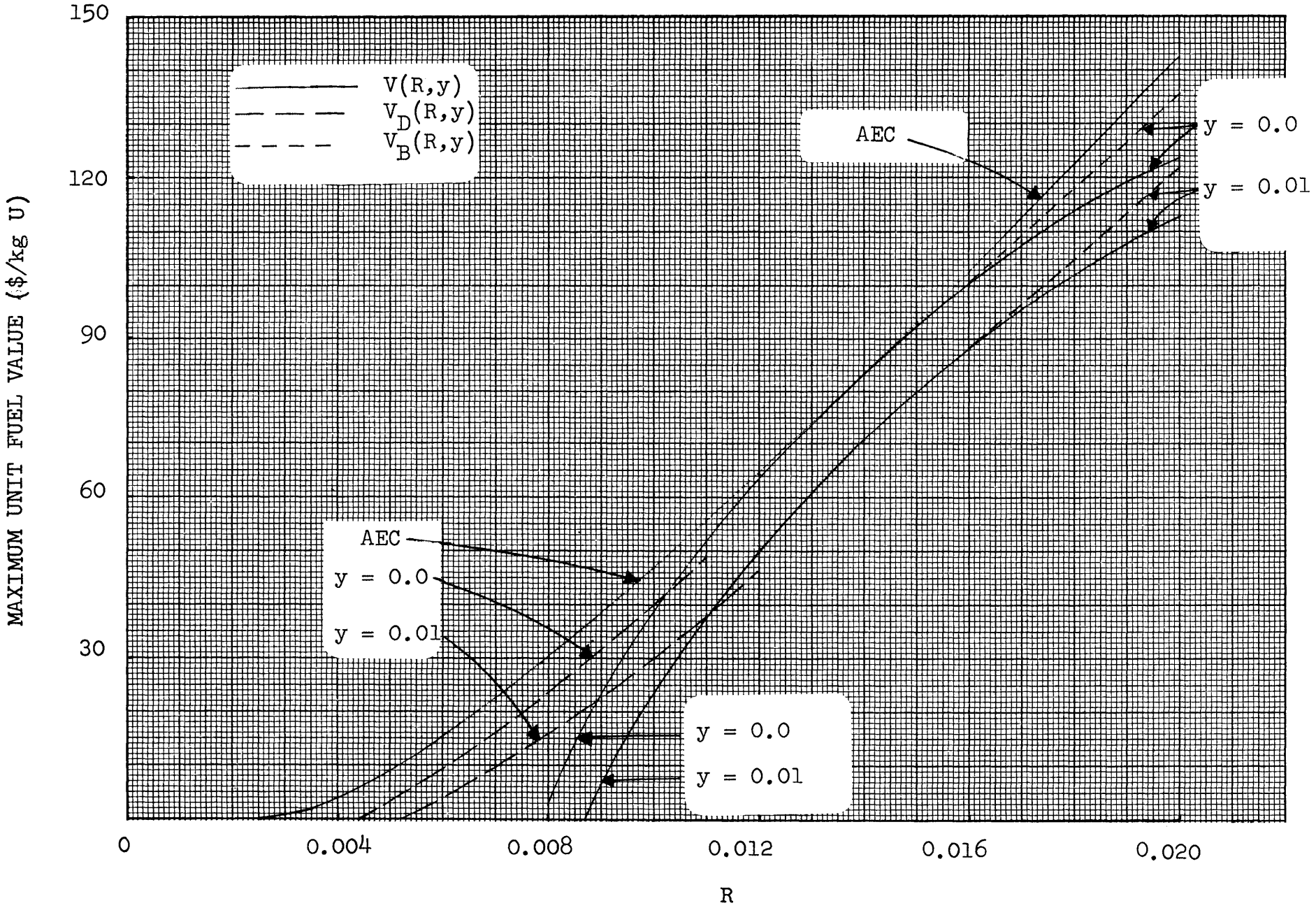


FIGURE VIII.1 The Effect of R and y on the Maximum Unit Fuel Value - $C_{U_3O_8}$ = \$8/lb, C_N = \$0/g. HWR.

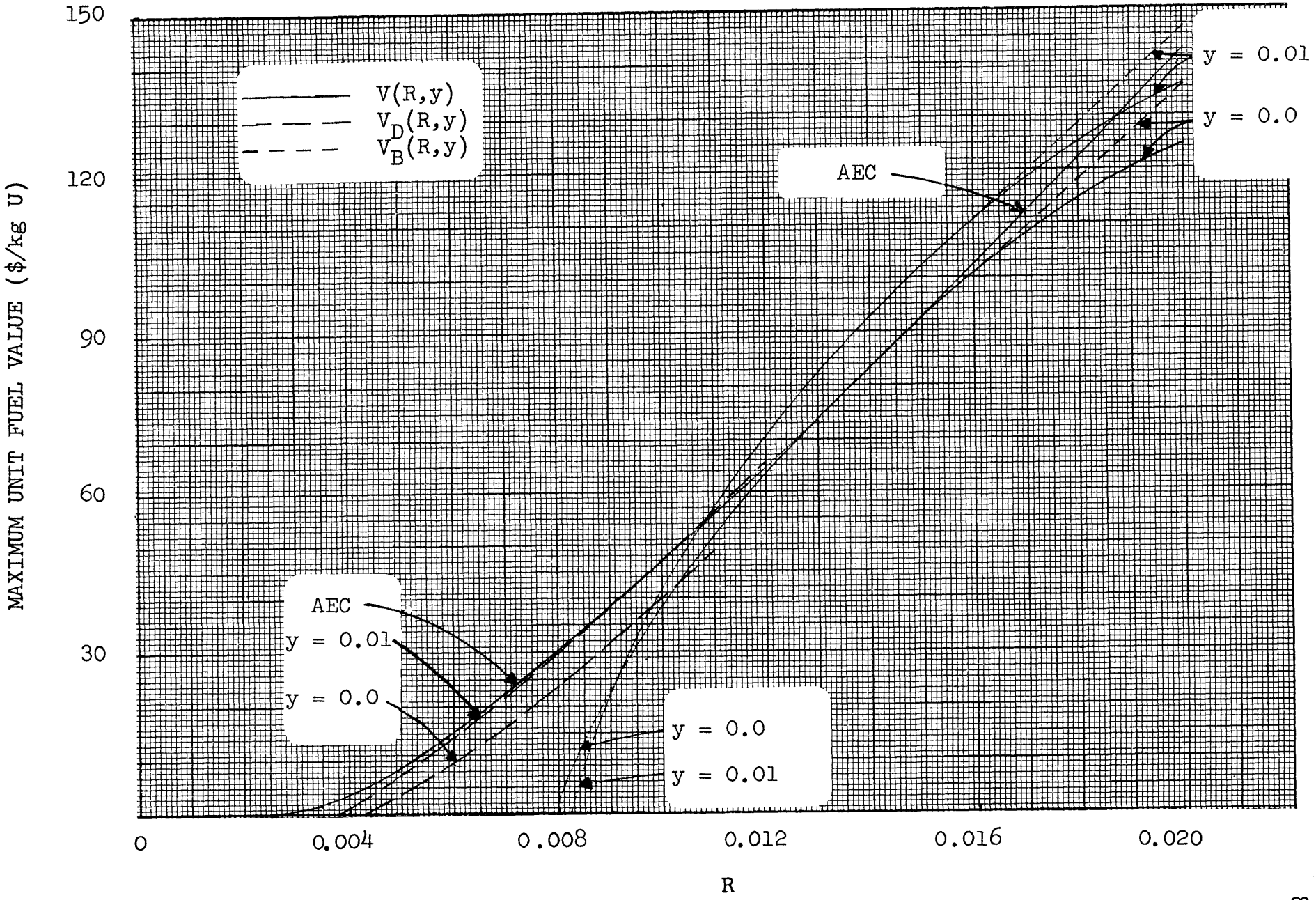


FIGURE VIII.2 The Effect of R and y on the Maximum Unit Fuel Value - $C_{U_3O_8}$ = \$8/lb, C_N = \$60/g. HWR.

Figure VIII.3 illustrates the effect of neptunium price and uranium enrichment on the maximum unit fuel value of uranium containing 0.01 U-236 weight fraction at a U_3O_8 price of \$8/lb. As can be seen, the maximum unit fuel value increases linearly with increasing neptunium price.

Figure VIII.4 illustrates the effect of U_3O_8 price and uranium enrichment on the maximum fuel value of uranium containing 0.01 weight fraction U-236, at a neptunium price of \$0/g. As can be seen, the uranium value increases nearly linearly with increasing U_3O_8 price.

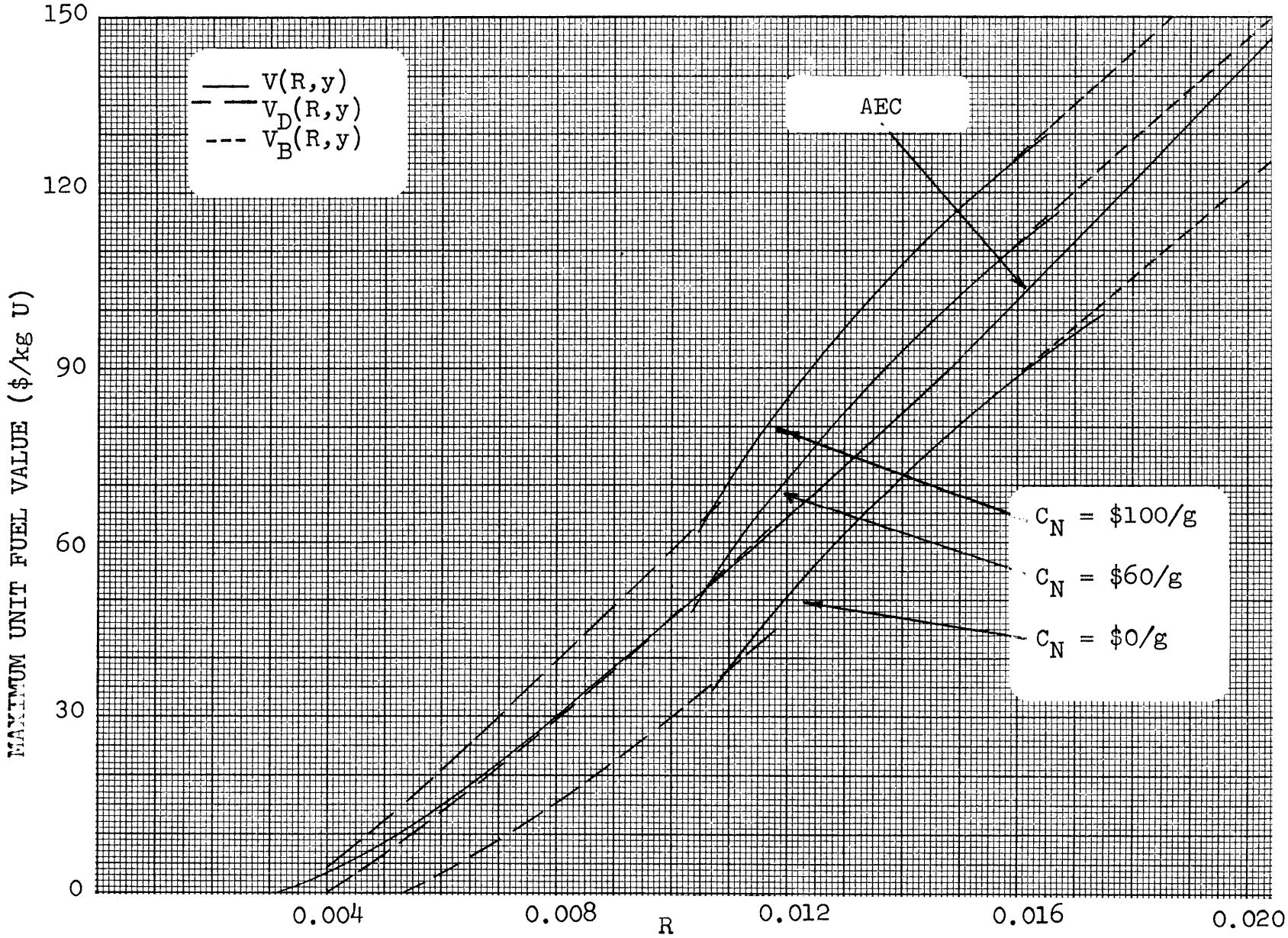


FIGURE VIII.3 The Effect of Neptunium Price on the Maximum Unit Fuel Value - HWR

$C_{U_3O_8}$ = \$8/lb y = 0.01

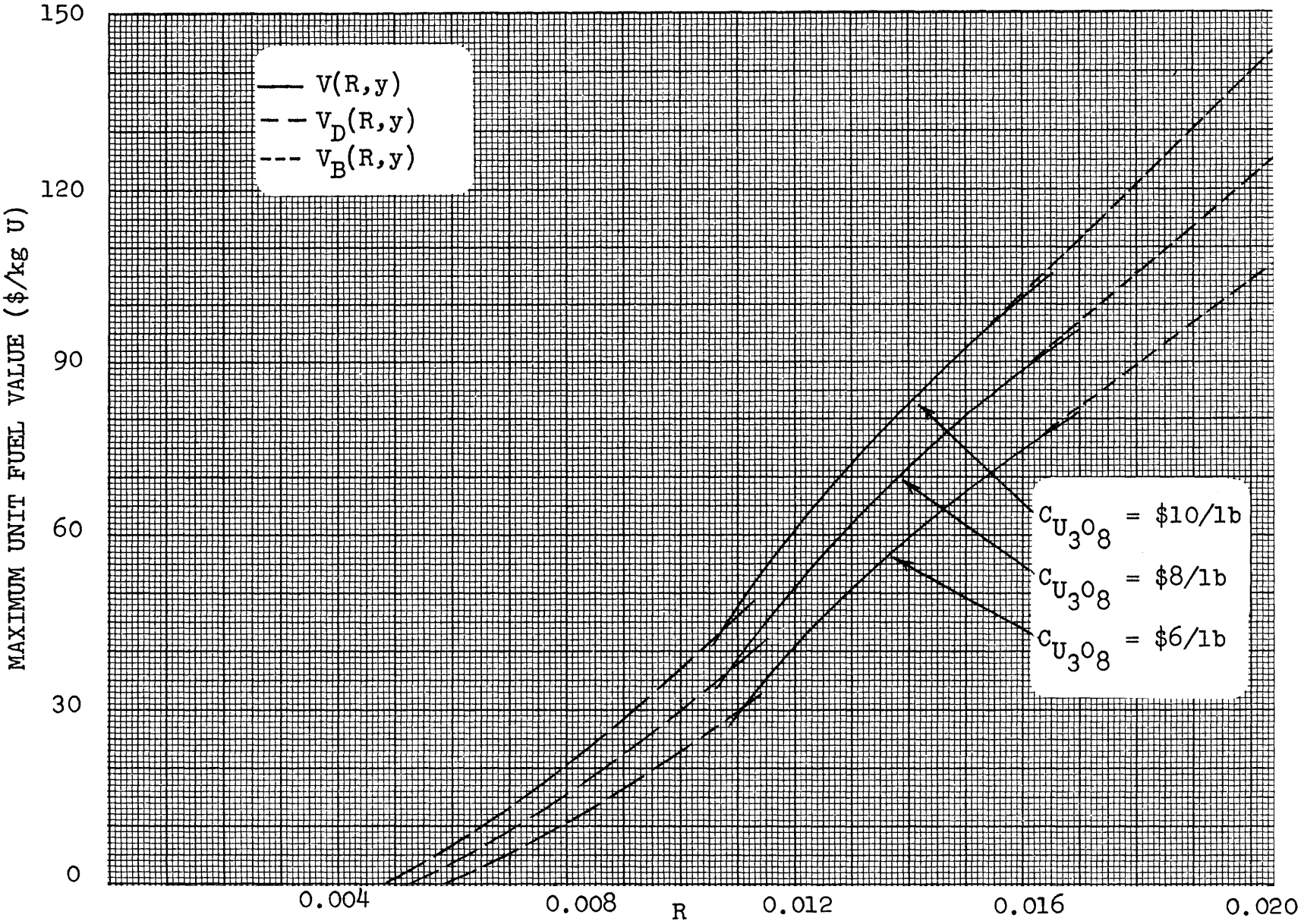


FIGURE VIII.4 The Effect of U_3O_8 on the Maximum Unit Fuel Value - HWR
C_N = \$0/g y = 0.01

In order to better characterize and describe the effect of U-236, a penalty for the presence of U-236 was defined as follows

$$\delta(R,y) = \frac{(1-y)V_m(R,0) - V_m(R,y)}{1000\ y} \left(\frac{\$}{g\ \ U\text{-}236}\right) \qquad \text{(VIII.1)}$$

The penalty is the reduction of value of (1-y) kilograms of uranium containing U-235 and U-238 in weight ratio R when y kilograms of U-236 are added to the mixture, per grams of U-236 added. This penalty gives a realistic measure of the effect of U-236 since the amounts of U-235 and U-238, the isotopes of principal fuel value, are held constant while a given amount of U-236 is added. Penalties for cases 5 and 7 where modified results have been obtained are listed in Table VIII.2. It is interesting to note immediately that for higher C_N (e.g. C_N = \$60/g) the penalties are negative which, of course, means that there is an economic advantage to having U-236 present in the uranium, thus producing greater quantities of Np-237. The U-236 penalties listed in Table VIII.2 have been calculated from the maximum fuel values listed at discrete values of R and y in Table VIII.1.

The U-236 penalty curves which have been plotted in Figure VIII.5 have been calculated from fuel value information from Table VIII.1.

TABLE VIII.2

U-236 Penalty, $\delta(R,y)$, $(\frac{\$}{g\ U\text{-}236})$. HWR.

	R = 0.006	0.008	0.010	0.012	0.014	0.016	0.018	0.020
Case 5: $C_{U_3O_8}$ = \$8/lb, C_N = \$0/g								
y = 0.005	0.69	0.90	1.05	1.46	1.24	1.24	1.25	1.25
0.01	0.58	0.77	0.93	1.31	1.17	1.15	1.19	1.21
0.02				1.13	1.01	.99	1.01	1.05
Case 7: $C_{U_3O_8}$ = \$8/lb, C_N = \$60/g								
y = 0.005	-0.35	-0.73	-0.87	-0.70	-1.18	-1.26	-1.30	-1.32
0.01	-0.47	-0.63	-0.76	-0.76	-1.03	-1.10	-1.16	-1.19
0.02			-0.64	-0.58	-0.87	-0.99	-1.04	-1.07

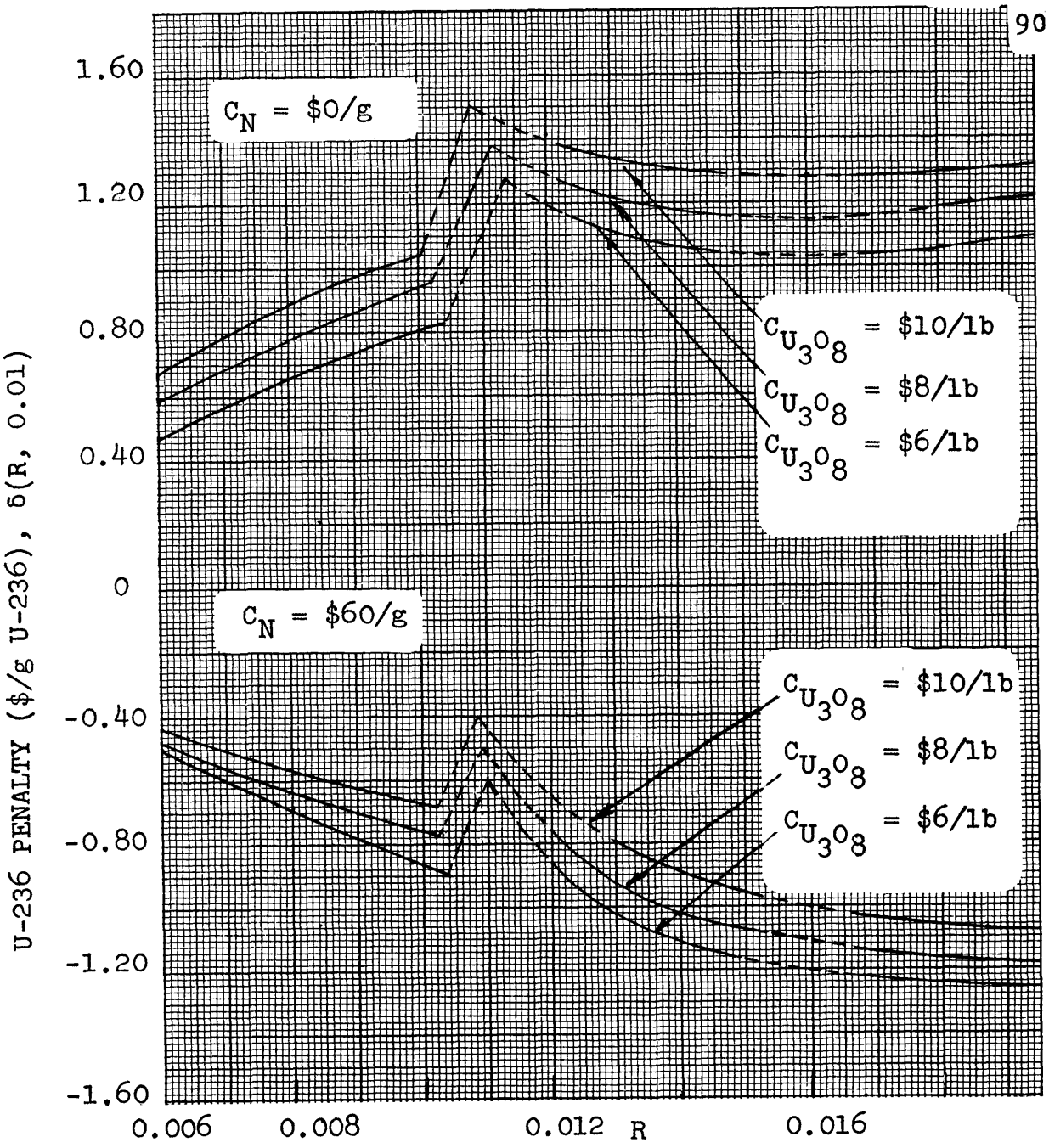


FIGURE VIII.5 The Effect of U_3O_8 and Np-237 Price on the U-236 Penalty - HWR
y = 0.01

It should be noted that the family of curves at the top of Figure VIII.5 correspond to C_N = \$0/g while those at the bottom correspond to C_N = \$60/g. It will be noted that the U-236 penalty is positive when neptunium has zero value, but is negative when the neptunium price is \$60/g. This means that U-236 is an economically desirable constituent of reactor feed at the latter neptunium price.

The shape of the curves and the origin of each segment will be explained in detail later. The dashed portion of the curves indicates areas of uncertainty originating from uncertainties in the fuel value curves of Figures VIII.1 and VIII.2. Since the tabulated penalties represents differences in discretely calculated fuel values, even small calculational errors in the determination of the fuel values would be expected to be noticeable when analyzing penalty data.

In order to examine the origin of the various segments of the penalty curve, Figure VIII.6 has been prepared for y = 0.01, $C_{U_3O_8}$ = \$8/lb, and C_N = \$0/g. The curves were obtained by defining the penalties as

$$\delta_1 = \frac{(1-y)V_D(R,0) - V_D(R,y)}{1000\ y},$$

$$\delta_2 = \frac{(1-y)V(R,0) - V(R,y)}{1000\ y},$$

and $$\delta_3 = \frac{(1-y)V_B(R,0) - V_B(R,y)}{1000\ y}.$$

The long dashed curve representing $V_D(R,y)$ in Figure

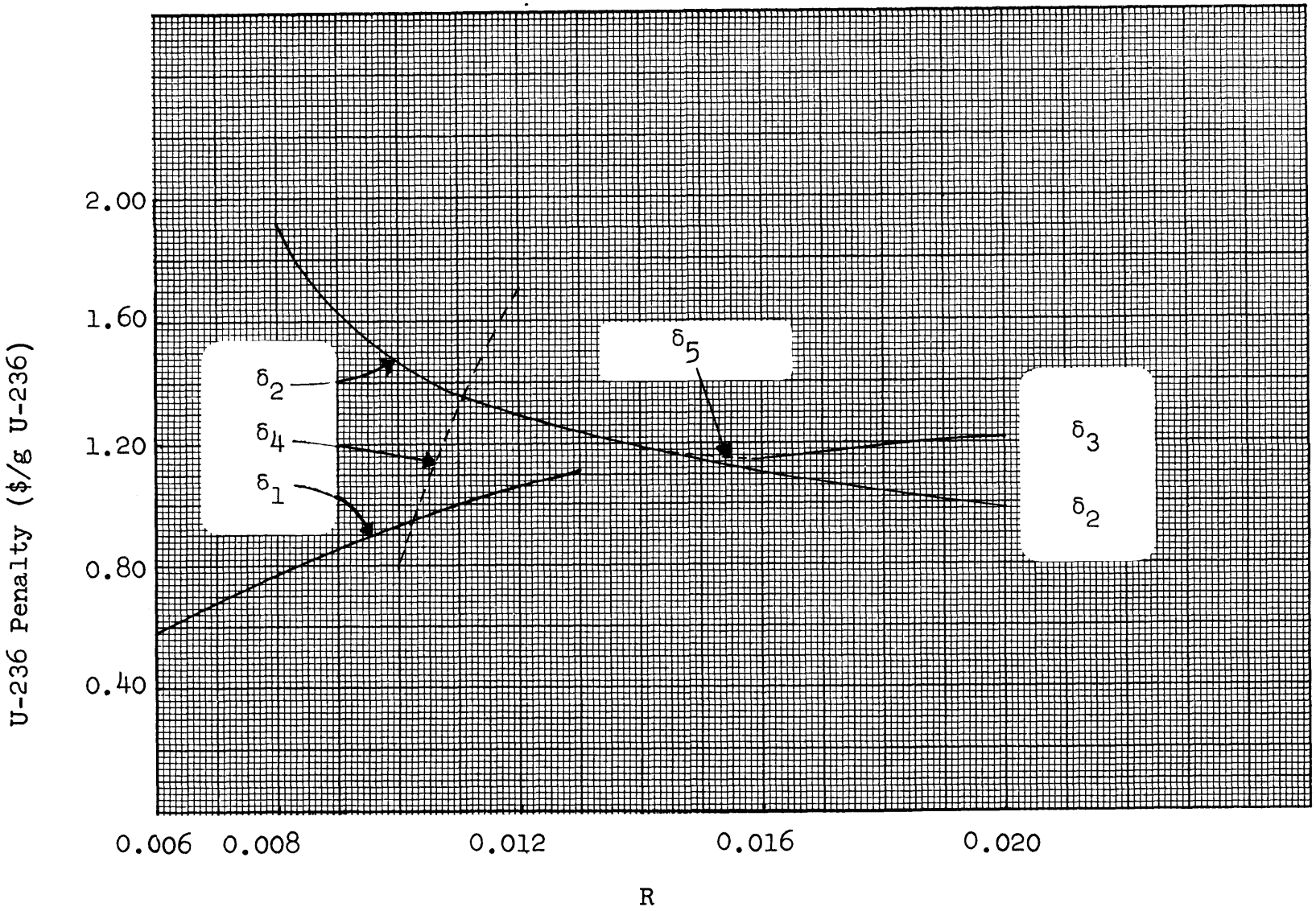


FIGURE VIII.6 Components of U-236 Penalty Curve for y = 0.01 - $C_{U_3O_8}$ = \$8/lb, C_N = \$0/g

VIII.1 crosses the solid curve representing V(R,y) at about R = 0.0102 for y = 0 and at about R = 0.0111 for y = .01; hence the region from R = 0.0102 to R = 0.0111 is the region of transition from curve δ_1 to curve δ_2. This dashed curve is labeled δ_4 and arises by defining

$$\delta_4 = \frac{(1-y)V(R,0) - V_D(R,y)}{1000\ y} .$$

The dashed curve δ_5 connects curves δ_2 and δ_3; the $V_B(R,y)$ curves in Figures VIII.1 and VIII.2 approach the V(R,y) curve almost tangentially as R approaches R* and therefore it is reasonable to assume a smooth intersection of curves δ_2 and δ_3. In order to determine $V_B(R,y)$ where R is close to R*, the flowrate, F_{nat}, of the blending material becomes very small and hence this becomes an unrealistic mode of operation.

Using the information in Figures VIII.1 and VIII.4, the penalty curve shown in Figure VIII.5 for y = .01 and C_N = \$0/g can be constructed. In the range from R = 0.006 to R = 0.0102, $V_D(R,y) = V_m(R,y) > V(R,y)$ for both y = 0 and y = 0.01 and $\delta = \delta_1$. In the range from R = 0.0102 to R = 0.0111, $V(R,y) = V_m(R,y) > V_D(R,y)$ for y = 0 but $V_D(R,y) = V_m(R,y) > V(R,y)$ for y = 0.01 and $\delta = \delta_4$. From R = 0.0111 to R = 0.014 unit fuel value data shows that $V(R,y) = V_m(R,y)$ and therefore $\delta = \delta_2$. In the range from R = 0.016 to R = 0.020

unit fuel value data shows that $V_B(R,y) = V_m(R,y)$ and therefore $\delta = \delta_3$. We also know that within the range R = 0.014 to R = 0.016 there has to be a transition and in this region we set $\delta = \delta_5$ where δ_5 merely connects the δ_2 and δ_3 curves. We have now completed the R range from 0.006 to 0.020 and have shown how the penalty curves in Figure VIII.5 were constructed as well as showing the reason for dashing the curves in the uncertain transition areas. In addition the difference in the slope of the transition part of the penalty curve can be understood when one realizes that the R value for the transition from $V_D(R,y) = V_m(R,y)$ to $V(R,y) = V_m(R,y)$ increases with increasing y. The "initial R" for transition (in our example R = 0.0102) is the same for all y because it results from $V_D(R,0) = V_m(R,0)$ changing to $V(R,0) = V_m(R,0)$ and since $V_m(R,0)$ is a key value in determining all the $\delta(R,y)$, this R point is the same for all $\delta(R,y)$ curves. However, the "final R" for transition (in our example R = 0.0111) occurs because $V_D(R,y) = V_m(R,y)$ changes to $V(R,y) = V_m(R,y)$ and since $V_m(R,y)$ is a key value in determining $\delta(R,y)$ and since the transition R is a function y, the "final R" will vary with y, as is shown in Figure VIII.5; with C_N = \$0/g, for y = 0.005 transition is complete when R = 0.0107, for y = 0.01 transition is

complete when R = 0.0111, and for y = 0.02 transition is complete when R = 0.0117.

If a reactor operator has penalty curves available and knows the y = 0 maximum fuel value curve for his fuel, he can determine the maximum value of fuel containing U-236 from the following expression

$$V_m(R,y) = (1-y)V_m(R,0) - 1000y\ \delta(R,y) \qquad \text{(VIII.2)}$$

The reactor operator could be expected to know $V_m(R,0)$ from standard design calculations but is less likely to have penalty curves determined for his particular reactor; therefore application of penalty curves derived in this study should give any operator of a heavy water moderated reactor an approximate value of fuel containing U-236.

Several interesting penalty results can be investigated at a given R = 0.014 at which $V(0.014,y) = V_m(0.014,y)$. At R = 0.014, $\delta(0.014,y)$ was calculated for y = 0.005, 0.01, 0.02, and 0.03 for the twelve base cases and the results are tabulated in Table VIII.3. These penalties are of signi-

cant interest because they have been calculated for R = 0.014 which is close to the optimum R for all cases which is the region a reactor operator is likely to operate within if uranium is purchased from the AEC.

The linearity of $\delta(0.014,y)$ with C_N was investigated by calculating the change in the U-236 penalty per \$/g change in Np-237 price; the results for the three Np-237 price intervals of \$0/g - \$20/g, \$20/g - \$60/g, and \$60/g - \$100/g are shown in Table VIII.4. For a given $C_{U_3O_8}$ and y, the U-236 penalty per \$/g change in Np-237 is essentially constant for the three intervals of C_N; thus $\delta(0.014,y)$ is in fact linear with C_N. Since the penalty values for a given y are practically the same for the three U_3O_8 prices and since there is only a slight change in penalty with $C_{U_3O_8}$, linear interpolation is justified for natural U_3O_8 price as well.

We can now define the indifference value for Np-237, C_N^o, as that value at which the U-236 penalty equals zero; at that value of Np-237 it is a matter of indifference whether one purchases U-235 plus U-238 free of U-236 or the same amount of U-235 plus U-238 containing U-236. With the known linearity $\delta(0.014,y)$

TABLE VIII.3

U-236 Penalty for R = 0.014, $\delta(0.014,y)$ $\left(\frac{\$}{g\ U\text{-}236}\right)$ in HWR.

Case	Natural U_3O_8 price, $C_{U_3O_8}$ ($/lb)	Np-237 price, C_N ($/g)	U-236 Penalty, $/g U-236, δ			
			y = 0.005	y = 0.01	y = 0.02	y = 0.03
1	10	0	1.34	1.26	1.08	1.01
2	10	20	0.54	0.53	0.46	0.47
3	10	60	-1.08	-0.93	-0.79	-0.63
4	10	100	-2.68	-2.40	-2.04	-1.72
5	8	0	1.24	1.17	1.01	.95
6	8	20	0.44	0.44	0.39	0.40
7	8	60	-1.18	-1.03	-0.87	-0.70
8	8	100	-2.78	-2.48	-2.11	-1.79
9	6	0	1.14	1.08	0.93	0.87
10	6	20	0.34	0.35	0.30	0.32
11	6	60	-1.27	-1.12	-0.95	-0.77
12	6	100	-2.88	-2.58	-2.20	-1.87

TABLE VIII.4

Change of U-236 Penalty with Neptunium Price in HWR

	$\frac{\Delta\delta}{\Delta C_N}$, $\frac{\$/gU\text{-}236}{\$/gNp\text{-}237}$			
	y = 0.005	y = 0.01	y = 0.02	y = 0.03
$C_{U_3O_8}$ = \$10/lb				
ΔC_N = \$20 - \$0	0.040	0.037	0.031	0.027
ΔC_N = \$60 - \$20	0.041	0.037	0.031	0.028
ΔC_N = \$100 - \$60	0.040	0.038	0.031	0.027
$C_{U_3O_8}$ = \$8/lb				
ΔC_N = \$20 - \$0	0.040	0.037	0.031	0.028
ΔC_N = \$60 - \$20	0.041	0.037	0.032	0.028
ΔC_N = \$100 - \$60	0.040	0.036	0.031	0.027
$C_{U_3O_8}$ = \$6/lb				
ΔC_N = \$20 - \$0	0.040	0.037	0.032	0.028
ΔC_N = \$60 - \$20	0.040	0.037	0.031	0.027
ΔC_N = \$100 - \$60	0.040	0.037	0.031	0.028

with C_N it is now a simple matter to calculate the indifference value of Np-237, C_N^o. These values are given in Table VIII.5 as a function of $C_{U_3O_8}$ and y. The increase in C_N^o with y occurs because as y increases, the increased production of Np-237 is insufficient to offset the decrease in reactivity caused by the poisoning effect of the U-236 (due to nonlinearity of the Np-237 production rate with y); therefore C_N^o increases as y increases. The other effect is that as the U_3O_8 price increases, C_N^o also increases. The conclusion that can be drawn is that for the present U_3O_8 price equal to \$8/lb, a Np-237 price in the range of \$30/g to \$35/g will lead to relative indifference on the part of the operator of a heavy water moderated reactor whether the uranium he purchases contains U-236 or whether it is U-236 free.

TABLE VIII.5

Indifference Value of Neptunium at R = 0.014

	C_N^o, \$/g Np-237			
	y = 0.005	y = 0.01	y = 0.02	y = 0.03
$C_{U_3O_8}$ = \$10/lb	33.30	34.50	34.70	37.10
$C_{U_3O_8}$ = \$8/lb	30.90	32.00	32.40	34.60
$C_{U_3O_8}$ = \$6/lb	28.40	29.50	29.60	31.80

IX. VALUE OF URANIUM AS FEED FOR PRESSURIZED WATER REACTOR WITH SPENT FUEL FED TO HEAVY WATER REACTOR

A. Pressurized Water Reactor Characteristics

The previous report on this project (1) utilized the CELL and MOVE codes to work out the fuel cycle characteristics of the 430 Mwe pressurized water reactor built by Westinghouse for the San Onofre station of the Southern California Edison Co. and San Diego Gas and Electric Co. Calculations were made for steady-state modified four-zone scatter refueling of UO_2 fuel with 24.3-mil zircaloy cladding. Principal characteristics of the reactor are summarized in Table B2 of Appendix B; more details are given in the previous report (1).

Table IX.1 restates from the previous report the fuel cycle performance of this reactor for eighteen combinations of R (U-235 to U-238 weight ratio in feed) and y (U-236 weight fraction in feed). For each of the 18 feed compositions studied, this table gives the isotopic content of spent uranium discharged from this pressurized water reactor in terms of R_S the U-235 to U-238 weight ratio and y_S the weight fraction of U-236.

B. Base Case Flow Scheme

In the previous report (1) this spent uranium was recycled either to fuel fabrication or through a diffusion plant in order to obviate the necessity of assigning a value to it. A different valuation procedure is used in

TABLE IX.1

Output from CELLMOVE - PWR

R	y	Burnup (MWD/T)	Reactor Feed Rate (kg U/day) F_R	Spent Uranium Discharge (kg U/day) $F_S/(1-L_{RU})$	Spent Uranium Composition		Fissile Plutonium Discharged (kg/day) $K/(1-L_{RP})$	Np-237 Discharged (kg/day) $N/(1-L_{RP})$
					R_S	y_S		
0.020	0	15,119	71.21	69.611	0.0089	0.0019	0.3482	0.01036
0.025	0	22,369	48.137	46.601	0.0091	0.0027	0.2830	0.01221
0.030	0	28,930	37.221	35.727	0.0095	0.0035	0.2458	0.01373
0.040	0	40,579	26.536	25.088	0.0109	0.0051	0.2047	0.01620
0.050	0	50,712	21.233	19.810	0.0125	0.0065	0.1831	0.01823
0.060	0	59,613	18.063	16.656	0.0146	0.0080	0.1705	0.01996
0.020	0.01	10,738	100,273	98.543	0.0112	0.0111	0.4159	0.05888
0.025	0.01	17,516	61.474	59.839	0.0113	0.0117	0.3307	0.05461
0.030	0.01	23,538	45.749	44.163	0.0119	0.0123	0.2856	0.05170
0.040	0.01	34,403	31.299	29.774	0.0135	0.0135	0.2359	0.04790
0.050	0.01	44,462	24.218	22.729	0.0152	0.0148	0.2073	0.04552
0.060	0.01	53,282	20.209	18.740	0.0173	0.0161	0.1909	0.04412
0.020	0.025	6,536	164.755	162.872	0.0139	0.0255	0.5052	0.12170
0.025	0.025	12,503	86.123	84.366	0.0141	0.0259	0.3925	0.11080
0.030	0.025	17,929	60.059	58.368	0.0148	0.0263	0.3351	0.10295
0.040	0.025	27,814	38.714	37.098	0.0167	0.0272	0.2740	0.09200
0.050	0.025	36,986	29.114	25.541	0.0188	0.0283	0.2399	0.08447
0.060	0.025	45,371	23.773	22.190	0.0211	0.0294	0.2188	0.07909

the present report. Here, this spent uranium is to be given the value it would have as uranium of the stated isotopic content fed to a heavy water reactor. The necessary values for uranium feed to a heavy water reactor have just been developed in Table VIII.1 of the previous section.

Figure IX.1 shows the flow scheme for using spent fuel from a pressurized water reactor as feed for a heavy water reactor.

C. Value of Spent Fuel in HWR

Table IX.2 gives the values of spent uranium of composition (R_S, y_S) from the PWR for the 18 PWR feed compositions (R, y) when used as feed to an HWR as in Figure IX.1. Spent fuel values are given for a natural uranium price of \$8/lb U_3O_8 and neptunium prices C_N of \$0 and \$60/g. These values were obtained by two-dimensional Lagrangian interpolation and extrapolation of the uranium values of cases 5 and 7 of Table VIII.1.

D. Fuel Cycle Cost Equation

The general equation for the fuel cycle cost C_E in mills per kwh in terms of the value of uranium feed V(R,y) and spent uranium $C_S(R_S,y_S)$ is given by:

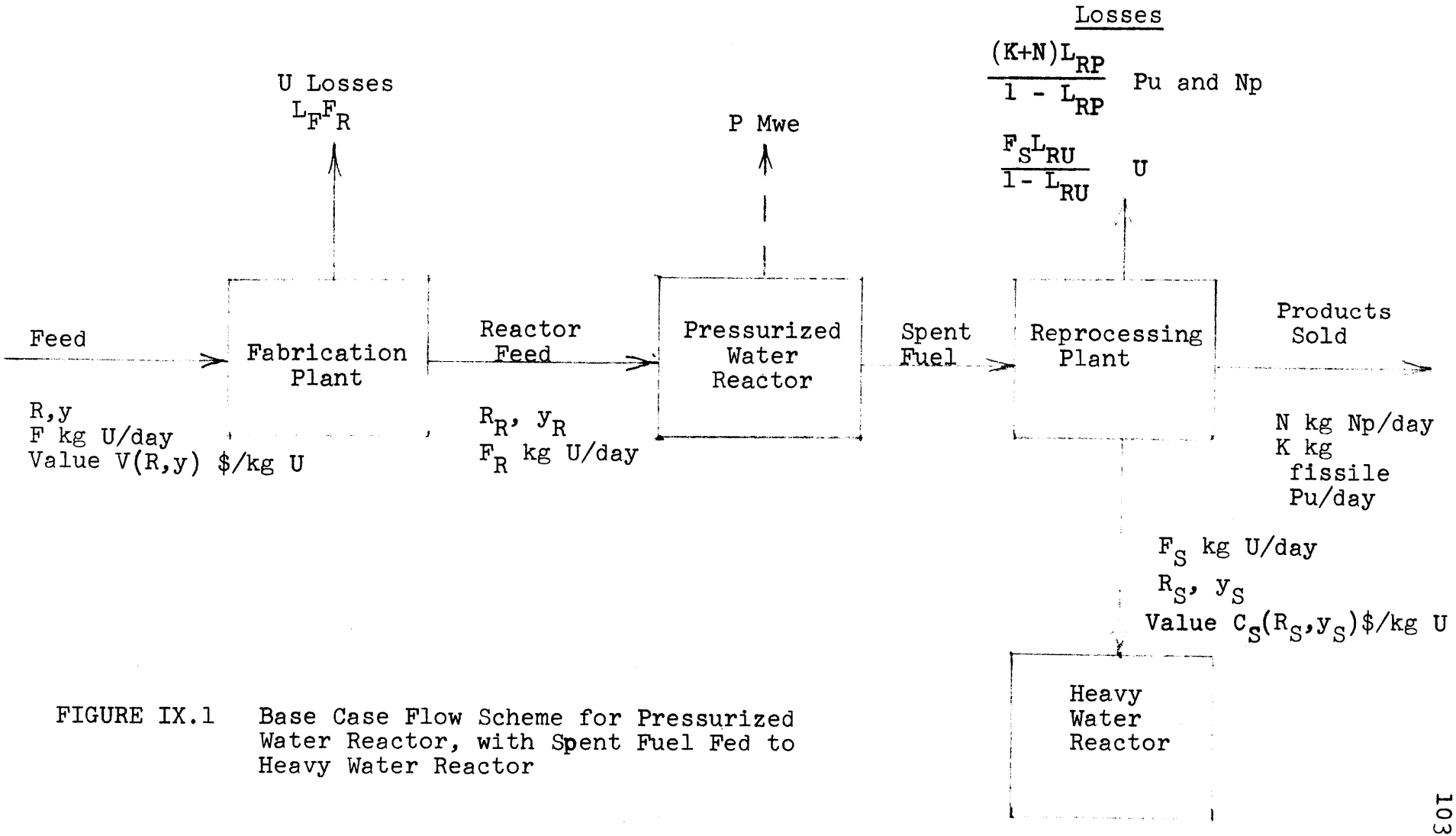


FIGURE IX.1 Base Case Flow Scheme for Pressurized Water Reactor, with Spent Fuel Fed to Heavy Water Reactor

TABLE IX.2

Composition of Spent Fuel From PWR and its Value as Feed in HWR

Feed to PWR		Spent Fuel From PWR		Value of Spent Uranium in HWR, $C_S(R_S,y_S)$. \$/kg U*	
U-235/U-238 Weight Ratio, R	Weight Fraction U-236, y	U-235/U-238 Weight Ratio, R_S	Weight Fraction U-236, y_S	C_N = \$0/g	C_N = \$60/g
0.020	0.000	0.0089	0.0019	29.05	32.62
0.025	0.000	0.0091	0.0027	29.81	34.91
0.030	0.000	0.0095	0.0035	32.17	38.89
0.040	0.000	0.0109	0.0051	43.09	52.92
0.050	0.000	0.0125	0.0065	60.10	74.34
0.060	0.000	0.0146	0.0080	78.53	97.04
0.020	0.010	0.0112	0.0111	39.50	60.79
0.025	0.010	0.0113	0.0117	40.12	62.52
0.030	0.010	0.0119	0.0123	46.76	71.00
0.040	0.010	0.0135	0.0135	62.49	90.14
0.050	0.010	0.0152	0.0148	76.84	107.57
0.060	0.010	0.0173	0.0161	93.21	127.50
0.020	0.025	0.0139	0.0255	56.08	100.35
0.025	0.025	0.0141	0.0259	57.70	102.81
0.030	0.025	0.0148	0.0263	64.02	110.73
0.040	0.025	0.0167	0.0272	80.08	129.97
0.050	0.025	0.0188	0.0283	95.87	149.17
0.060	0.025	0.0211	0.0294	111.39	174.89

*Based on \$8/lb U_3O_8 for natural uranium

24 PL C_E (cost of electricity, \$/day) =

$$F\ V(R,y)$$ value of net feed

$$+ F_R C_F$$ cost of fabrication

$$+ \left(\frac{F_S}{1-L_{RU}} + \frac{(K+N)}{1-L_{RP}}\right)(C_A + C_{SH})$$ cost of reprocessing and shipping

$$- 1000\ K\ C_K$$ credit for plutonium

$$- F_S\ C_S(R_S,y_S)$$ credit for spent uranium

$$- 1000\ N\ C_N$$ credit for neptunium

$$+ i\ t_F\left(\frac{V(R,y)}{1 - L_F} + C_F\right) F_R$$ interest during fabrication

$$+ i\ t_{RU}\left(C_S(R_S,y_S) - \frac{(C_A+C_{SH})}{1-L_{RU}}\right)F_S$$ interest on uranium inventory during reprocessing

$$+ i\ t_{RP}\left(1000\ K\ C_K + 1000\ N\ C_N - \frac{(N+K)}{1-L_{RP}}\ (C_A+C_{SH})\right)$$ interest on Pu and Np inventory during reprocessing

$$+ \frac{i\ I}{2 \times 365}\left[\frac{V(R,y)}{1 - L_F} + C_F + \frac{1000\ KC_K + 1000\ NC_N + F_S C_S(R_S,y_S)}{F_R} - \left(\frac{F_S}{1-L_{RU}} + \frac{(N+K)}{1-L_{RP}}\right)\frac{(C_A + C_{SH})}{F_R}\right]$$ interest on mean value of reactor inventory

(IX.1)

The fuel cycle equation given above is for the PWR for the base case mode of operation illustrated in Fig. IX.1.

In this equation,

P	is the net power output of the PWR reactor plant, Mw(e)
L	is the load factor
V(R,y)	is the value of uranium feed of composition R,y, \$/kg U
C_F	is the unit cost of fabrication, \$/kg U leaving fabrication plant. This price includes the cost of converting UO_3 or UF_6 into UO_2 in the case of the PWR and UO_3 or UF_6 into UC in the case of the HWR
C_{SH}	is the unit cost of shipping irradiated fuel, \$/kg fuel shipped
C_A	is the cost of reprocessing fuel, \$/kg of fuel entering the reprocessing plant. This price includes the cost of converting UNH to UO_3
C_K	is the credit received for plutonium, \$/g fissile plutonium
C_N	is the credit received for neptunium, \$/g neptunium
$C_S(R_S,y_S)$	is the credit received for spent uranium of composition R_S,y_S, \$/kg U

t_F	is the average pre-reactor fuel holdup time, years
t_{RU}	is the average post-reactor holdup time for spent uranium, years
t_{RP}	is the average post-reactor holdup time for neptunium and plutonium, years
i	is the fixed charge rate on working capital, yr^{-1}
I	is the total initial uranium loading of the reactor, kg uranium
L_F	is the fractional loss of uranium during fabrication, based on the material leaving the fabrication plant
L_{RU}	is the fractional loss of uranium during the reprocessing operation, based on the material entering the reprocessing plant
L_{RP}	is the fractional loss of neptunium and plutonium during the reprocessing operation, based on material entering the reprocessing plant

E. Minimum Fuel Cycle Cost

As was stated previously, the first step in calculating the value of uranium containing U-236 is to calculate the minimum fuel cycle cost using uranium free of U-236 priced on the AEC scale. This is done by solving the fuel cycle cost equation, Eq. (IX.1), for the net fuel cycle cost, C_E, with V(R,y) replaced by $C_{AEC}(R)$:

$$C_E = \frac{1}{24\ PL} \left\{ F\ C_{AEC}(R) + F_R\ C_F \right.$$

$$+ \left(\frac{F_S}{1 - L_{RU}} + \frac{(K+N)}{1 - L_{RP}}\right)(C_A + C_{SH}) - 1000\ K\ C_K$$

$$- F_S\ C_S(R_S, y_S) - 1000\ N\ C_N + 1\ t_F\left(\frac{C_{AEC}(R)}{1 - L_F} + C_F\right) F_R$$

$$+ 1\ t_{RU}\left(C_S(R_S, y_S) - \frac{(C_A + C_{SH})}{1 - L_{RU}}\right) F_S + 1\ t_{RP}\ (1000\ K\ C_K$$

$$+ 1000\ N\ C_N - \frac{(N+K)}{1-L_{RP}}(C_A + C_{SH})) + \frac{1\ I}{2 \times 365} \left[\frac{C_{AEC}(R)}{1 - L_F} \right.$$

$$+ C_F + \frac{1000\ K\ C_K + 1000\ N\ C_N + F_S C_S(R_S, y_S)}{F_R}$$

$$\left. \left. - \left(\frac{F_S}{1 - L_{RU}} + \frac{(N+K)}{1 - L_{RP}}\right) \frac{(C_A + C_{SH})}{F_R} \right] \right\} \qquad \text{(IX.2)}$$

The fuel cycle cost, C_E, is calculated at specified points over a wide range of enrichments, R, and the minimum fuel cycle cost, C_E^*, and the corresponding optimum enrichment, R^*, are calculated from these values of C_E either by interpolation methods or graphically.

The minimum fuel cycle cost C_E^* and the corresponding optimum enrichment R^* for the PWR with spent fuel valued from Table IX.2 as if fed to an HWR, calculated from Eq. (IX.2), are given in the last two columns of Table IX.3. The two middle columns give corresponding data for the PWR with spent uranium recycled through a diffusion plant, the procedure now practiced. These data for recycle through a diffusion plant were obtained in the previous report (1).

TABLE IX.3

Summary of Minimum Fuel Cycle Costs - PWR

		Recycle through Diffusion Plant		Spent U to HWR	
$C_{U_3O_8}$ ($/lb)	C_N ($/g Np)	R^*	C_E^* (mills/kwh)	R^*	C_E^* (mills/kwh)
8	0	0.0309	1.614	0.0315	1.526
8	60	0.0315	1.429	0.0320	1.430

It is noteworthy that the fuel cycle cost for the PWR would be almost 0.1 mills/kwh lower if it were possible to use its spent uranium as feed for a heavy water reactor, if neptunium had zero value. If the neptunium price were \$60/g, the minimum fuel cycle cost is almost the same for the two ways of using spent uranium.

When $C_N = 0$, the fuel cycle cost when spent uranium is fed to an HWR is lower than when the spent uranium is recycled through a diffusion plant because none of the U-236 made in the PWR returns to that reactor when spent fuel goes to an HWR, whereas some U-236 builds up in the PWR when spent fuel is recycled through a diffusion plant. When neptunium has zero value, this U-236 increases fuel cycle costs because of its effect as a neutron absorber. When C_N = \$60/g, the buildup of U-236 has little effect because the value of the additional neptunium produced from the added U-236 about compensates for the loss of neutrons.

F. Fuel Value Calculation

1. Base Case

Once the minimum fuel cycle cost C_E^* is calculated using uranium with no U-236, priced on the AEC scale, it is a simple matter to calculate the value of uranium used as feed in the base case mode. To do this, Eq. (IX.1) is used where the fuel cycle cost C_E is now replaced by the minimum fuel cycle cost C_E^*. Upon rearranging Eq. (IX.1) we obtain:

$$V(R,y) = \frac{1}{F + \frac{i\, t_F F_R}{1 - L_F} + \frac{1\, I}{2x365(1-L_F)}} \left\{ 24\, PL\, C_E^* - F_R C_F \right.$$

$$- \left(\frac{F_S}{1-L_{RU}} + \frac{(K+N)}{1-L_{RP}}\right)(C_A + C_{SH}) + 1000\, K\, C_K + F_S C_S(R_S, y_S)$$

$$+ 1000\, N\, C_N - i\, t_F\, C_F\, F_R - i\, t_{RU}\left(C_S(R_S, y_S) - \frac{(C_A + C_{SH})}{1-L_{RU}}\right) F_S$$

$$- i\, t_{RP}\left(1000\, K\, C_K + 1000\, N\, C_N - \frac{(N+K)}{1-L_{RP}}(C_A + C_{SH})\right)$$

$$- \frac{1\, I}{2x365}\left[C_F + \frac{1000\, K\, C_K + 1000\, N\, C_N + F_S C_S(R_S, y_S)}{F_R} \right.$$

$$\left. \left. - \left(\frac{F_S}{1-L_{RU}} + \frac{(N+K)}{1-L_{RP}}\right) \frac{(C_A + C_{SH})}{F_R} \right] \right\} \qquad \text{(IX.3)}$$

Using the above equation and values of $C_S(R_S, y_S)$ from Table VIII.1, the value of uranium can be calculated over a wide range of R and y.

2. Pre-Enrichment by Diffusion

Equation IX.3 gives the value of uranium V(R,y) when used as feed directly to a PWR, as indicated in Fig. IX.1. When the enrichment of this uranium is appreciably less than the optimum enrichment R* (at which the fuel cycle cost for feed free of U-236 is a minimum), a higher value for this uranium can be obtained by pre-enriching it by gaseous diffusion to an enrichment R_D near R* before feeding it to the PWR, as was done for the HWR in Fig. V.2. After enriching this uranium to composition R_D, y_D its value $V(R_D, y_D)$ is known because it is then used as feed to the base case for which the value has been determined by Eq. (IX.3). From $V(R_D, y_D)$ determined in this way, the value of fuel being pre-enriched by diffusion $V_D(R,y)$ may be calculated by Eq. (VI.4).

3. Blending with Natural Uranium

When the enrichment of uranium fuel is appreciably greater than the optimum enrichment R* (at which the fuel cycle cost for feed free of U-236 is a minimum), a higher value can be obtained for this uranium by blending with natural uranium to an enrichment R_B near R* before feeding it to the PWR, as was done for the HWR in Fig. V.3. After blending this uranium to composition R_B, y_B its value $V(R_B, y_B)$ is known because it is then used as feed to the base case for which the value has been determined by Eq. (IX.3). From $V(R_B, y_B)$ determined in this way, the value of blended fuel $V_B(R,y)$ may be calculated by Eq. (VI.5).

G. Base Case Uranium Fuel Values

1. Spent Uranium from PWR Fed to HWR

Uranium fuel values for the base case mode of operation calculated from Eq. (IX.3) are tabulated in Appendix G, Table G.1 and are illustrated graphically in Figure IX.2 for a \$0/g neptunium price. V(R,y) is shown as a function of enrichment R for three weight fractions y of U-236. It is of interest to discuss the general features of this base case curve. As can be seen, the y=0 fuel value curve is tangent to the AEC price scale curve at the optimum enrichment R^*. This is a necessary consequence of the method used to determine the fuel value. The basic valuational principle states

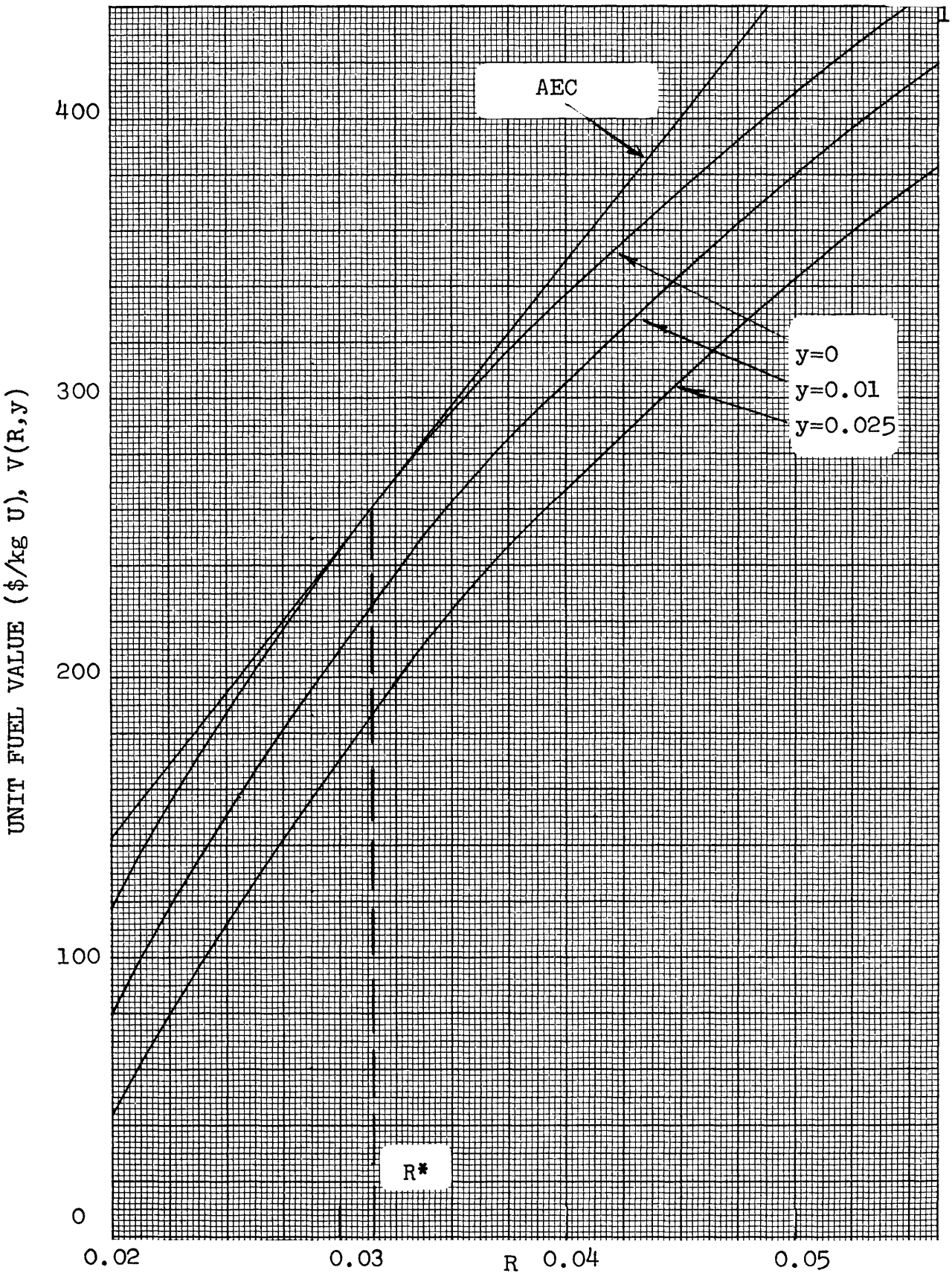


FIGURE IX.2 The Effect of R and y on Base Case Unit Fuel Value - PWR
$C_{U_3O_8}$ = \$8/lb C_N = \$0/g

that the total fuel cycle cost, C_E, using fuel of composition R,y must be equal to the minimum fuel cycle cost, C_E^*, using fuel free of U-236, purchased as UF_6 on the AEC scale, and of the optimum enrichment, R^*. Thus, it is expected that the y=0 curve would be tangent to the AEC price scale curve at the optimum enrichment and lie below it at all other values of R. (It must be remembered that the cost of converting UF_6 to UO_2 was assumed to be equal to the cost of converting UO_3 to UO_2.) It can also be seen in this neptunium equals \$0/g curve that the fuel value decreases with increasing amounts of U-236 and that the effect of a given amount of U-236 decreases as the amount of U-236 increases.

The base case results for a neptunium price of \$60/g are shown in Fig. IX.3. These curves show many of the same characteristics as the \$0/g curves, the big difference being that the presence of U-236 now increases the value of the fuel. Several other characteristics are also of note. At low values of R and the resulting low burnups, the effect of U-236 as a thermal poison overrides the increase in value of the fuel due to the buildup of neptunium. This is why the fuel of composition y=0.01 is less valuable than fuel of composition y=0 at R equals 0.02. As the enrichment and

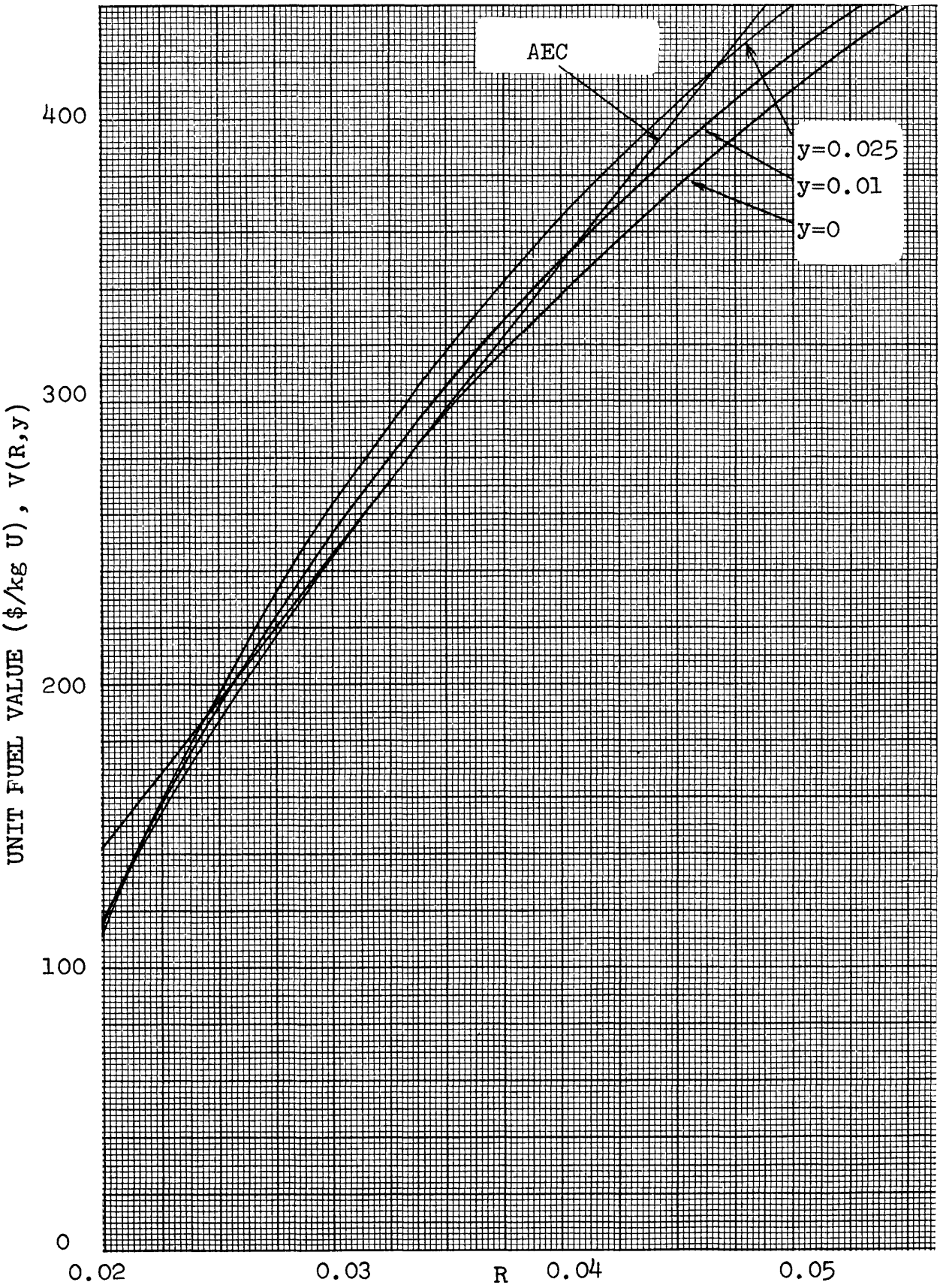


FIGURE IX.3 The Effect of R and y on Base Case Unit Fuel Value - PWR
$C_{U_3O_8}$ = \$8/lb C_N = \$60/g

corresponding burnup increase, the increased value due to the buildup of neptunium predominates over the effect of U-236 as a poison and the trend reverses itself, the fuel of higher weight fraction U-236 now being the more valuable. This trend, however, cannot continue indefinitely, for at some high y value the poisoning effect, causing decreased reactivity lifetime, will override the increase in neptunium credit.

2. Comparison with Other Fuel Cycle Schemes

A comparison of the PWR base case curves when spent uranium is fed to an HWR and when it is recycled through a diffusion plant (1) reveals the basic differences between the two fuel cycle schemes. Figure IX.4 compares the curves for y equals 0 and 0.01 for a neptunium value of $0/g. Figures IX.5 and IX.6 compare similar curves for a neptunium value of $60/g. The y=0.01 curves for the two fuel cycle schemes best illustrate the basic differences between the two systems. When the neptunium price equals $0/g, the value of the fuel being fed to the reactor using a recycle to diffusion fuel cycle is much less than if the fuel were fed to the same reactor with spent fuel going to an HWR. In the recycle to diffusion fuel cycle, the U-236, which is a thermal poison, is being recycled through the reactor. Each time it passes through the diffusion plant, it is concentrated in the heads stream, i.e., the stream which is recycled to the reactor.

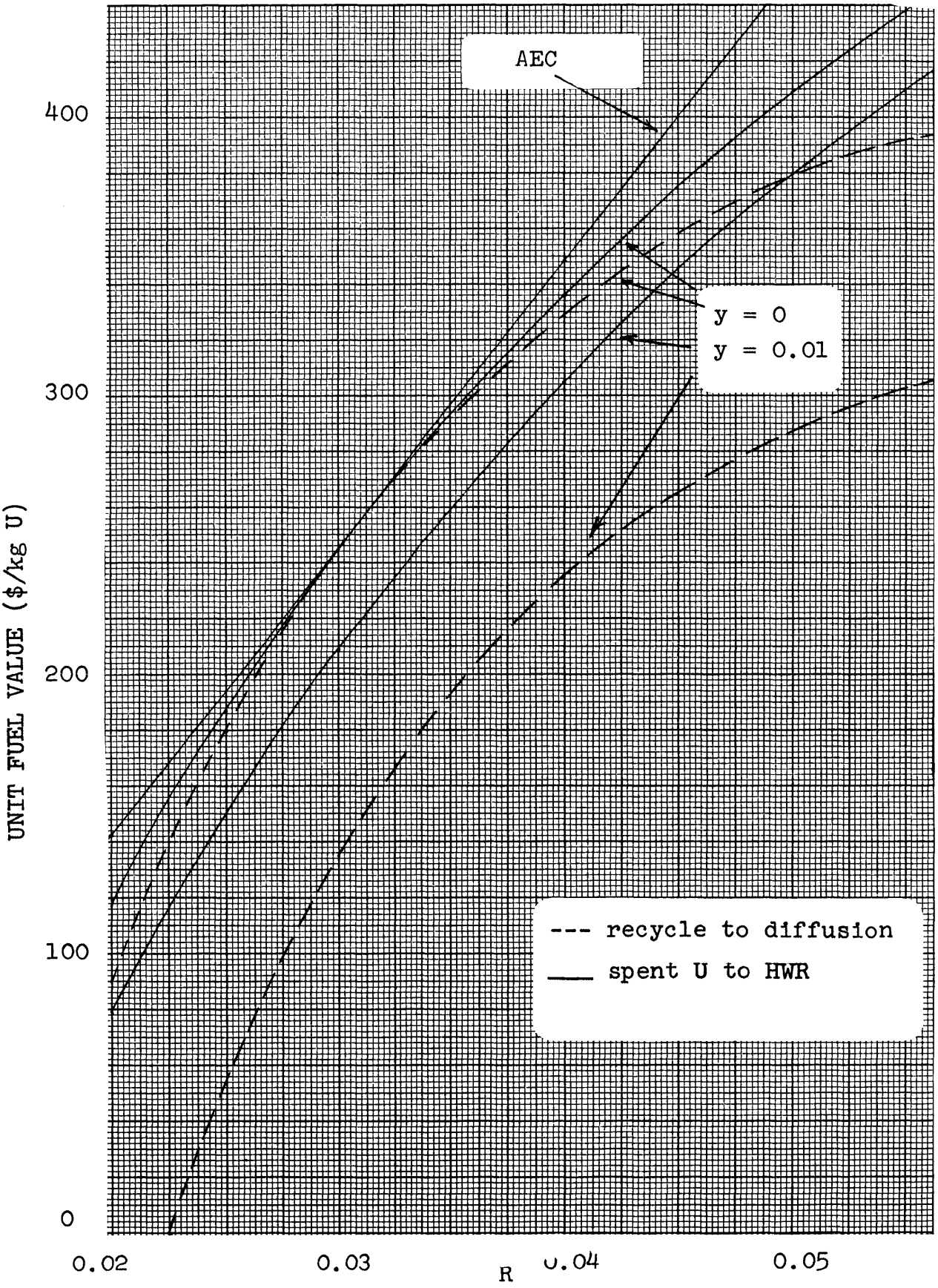


FIGURE IX.4 Comparison of the Base Case Fuel Value for PWR with Spent Fuel Going to HWR and with Spent U Recycled through Diffusion Plant
$C_{U_3O_8}$ = \$8/lb C_N = \$0/g

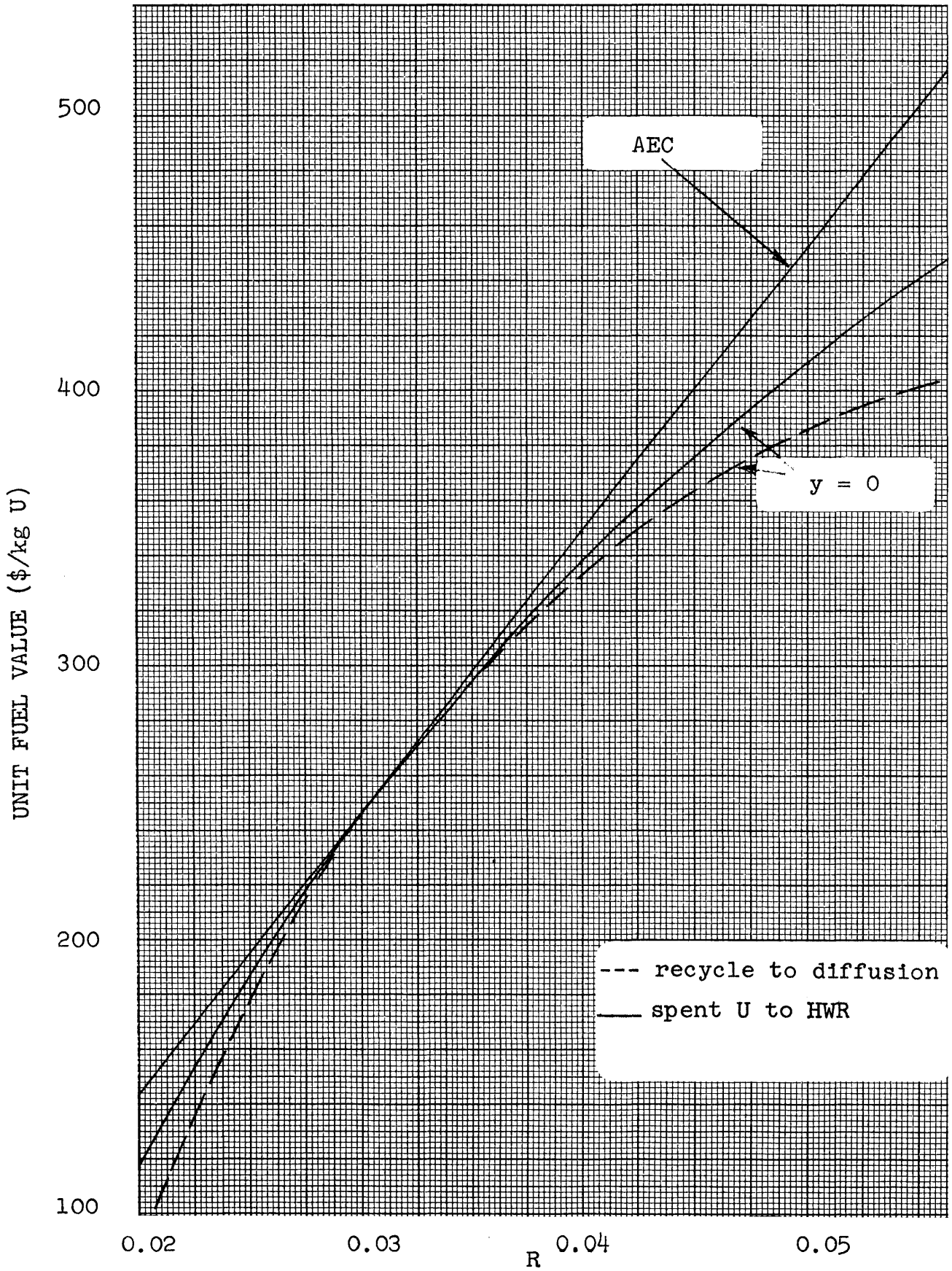


FIGURE IX.5 Comparison of the Base Case Fuel Value for PWR with Spent Fuel Going to HWR and with Spent U Recycled through Diffusion Plant
$C_{U_3O_8}$ = \$8/lb C_N = \$60/g

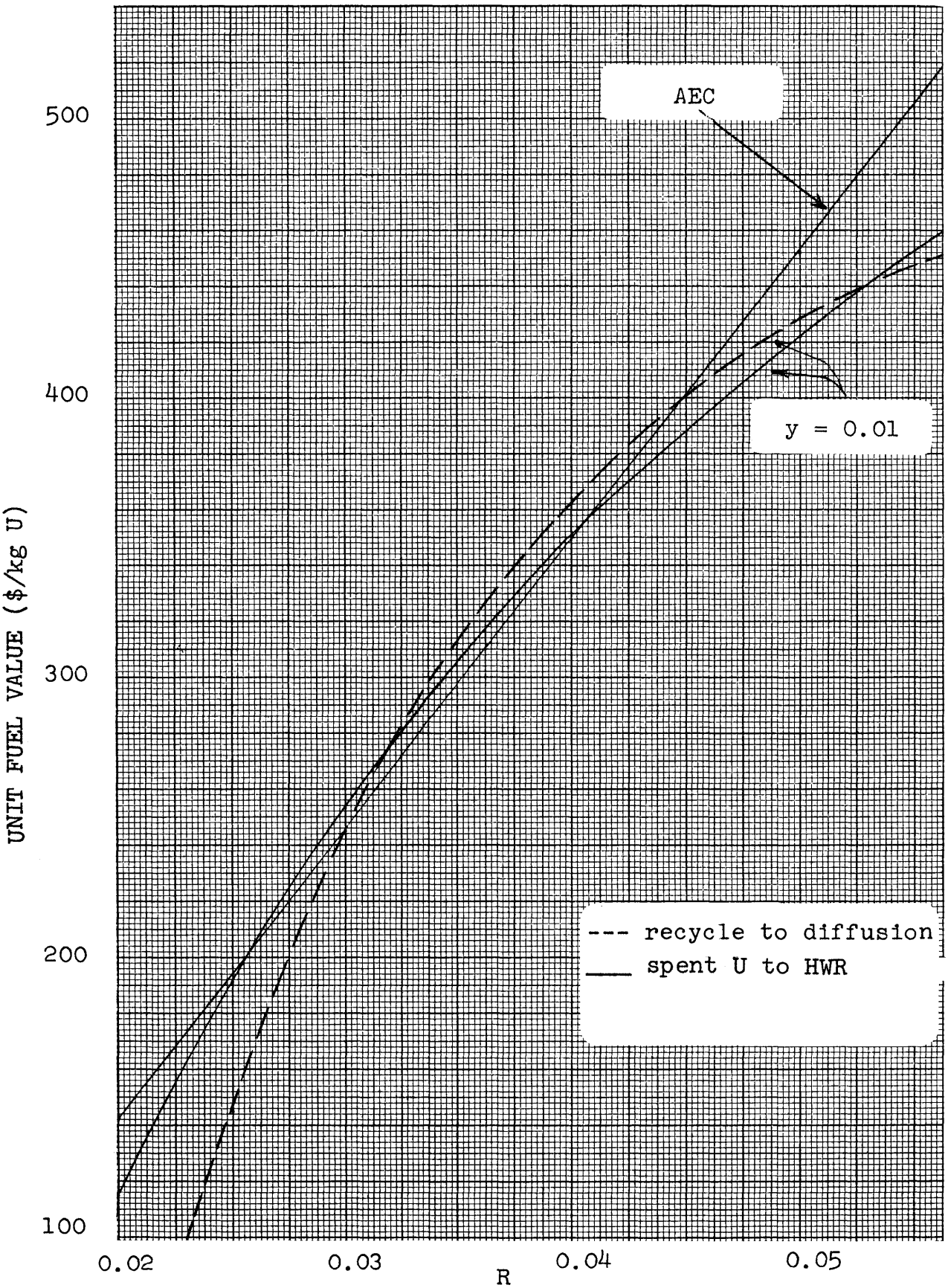


FIGURE IX.6 Comparison of the Base Case Fuel Value for PWR with Spent Fuel Going to HWR and with Spent U Recycled through Diffusion Plant
$C_{U_3O_8}$ = \$8/lb C_N = \$60/g

Thus, although the new fuel being charged to the system has a y value equal to 0.01, the actual concentration of U-236 fed to the reactor is much higher. In the discharge to HWR cycle the concentration of U-236 entering the reactor is exactly 0.01. The reason why these differences are not as apparent in the y=0 case, especially near the optimum enrichments, is due solely to the valuational principle used to determine the effect of U-236 on fuel value. This valuational principle states that fuel of composition R,y is to have a value such that the net fuel cycle cost with this uranium feed is equal to the overall fuel cycle cost <u>for the same fuel cycle</u> with uranium containing no U-236, priced on the existing AEC scale, and operated at the feed enrichment which gives minimum fuel cycle cost. Thus, even though the fuel cycle cost of the recycle to diffusion fuel cycle is higher than the other cycle, the fuel values are very nearly the same near the optimum enrichments, since the fuel value curves are tangent to the AEC scale at these points by definition. It is only at $y > 0$ that it becomes apparent that recycle to diffusion is more sensitive to U-236 than the discharge to HWR cycle. This is especially apparent when comparing the y=0.01 fuel value curves for the two neptunium prices. When going from a \$0/g neptunium price to a \$60/g price, the fuel value for the fuel to HWR cycle increases on the average \$45/kg U,

where in the recycle to diffusion fuel cycle the increase is approximately \$140/kg U.

H. Maximum Fuel Value

1. Spent Uranium from PWR Fed to HWR

As was stated earlier, under certain circumstances, the base case mode of operation is not the most advantageous fuel cycle scheme. If the uranium to be fed to the PWR is of an enrichment much lower than the optimum enrichment, R^*, the best use can be made of this fuel if it is first enriched in a diffusion plant before being fed to the reactor. This mode of operation is called the pre-enrichment by diffusion mode and the uranium value so obtained is $V_D(R,y)$. If the fuel fed to the reactor is of an enrichment much greater than the optimum enrichment then the best mode of operation is the blending with natural uranium mode and the fuel value so obtained is $V_B(R,y)$. In this report the base case values, $V(R,y)$, were calculated over the entire range of enrichments; the pre-enrichment by diffusion values, $V_D(R,y)$, were calculated at the enrichments less than the optimum enrichment; and the blending with natural uranium values, $V_B(R,y)$, at enrichments greater than the optimum. Values of V_D are given in Table G.2 of Appendix G; values of V_B are in Table G.3.

The maximum unit fuel value $V_m(R,y)$ is defined as the greater of $V(R,y)$ and $V_D(R,y)$ in the region of lower enrichment ($R < R^*$) and the greater of $V(R,y)$ and $V_B(R,y)$ in the

region of higher enrichment ($R > R^*$). According to the definition of the basic economic principle, $V_m(R,y)$ then is the maximum price that can be paid for fuel of composition R,y without increasing the cost of generating electricity above that incurred when using fuel free of U-236, of the optimum enrichment, priced on the AEC scale. Maximum fuel values obtained thus from the data of Appendix G are given in Table IX.4.

Figure IX.7 shows the maximum fuel value curve for neptunium equals \$0/g. Looking at the y=0.01 curve, the long-dashed line represents that part of the curve where pre-enrichment by diffusion is the best mode of operation. At about R equals 0.025 the base case (solid line) mode becomes the most advantageous mode of operation, i.e., gives the maximum fuel value. This is true up to an enrichment of 0.038. At this point and at enrichments greater than this, blending with natural uranium (short-dashed line) gives the maximum fuel value. $V_m(R,y)$ then is a composite of three curves, $V_D(R,y)$, $V(R,y)$, and $V_B(R,y)$. At the cross over points, R=0.025 and R=0.038, the curves were extended a bit to illustrate this point.

Figure IX.8 is a similar plot of the maximum fuel value for C_N = \$60/g.

TABLE IX.4

Maximum Unit Fuel Values in PWR, with Spent Uranium Credited as Fuel in HWR, $/kg U

R =		0.015	0.020	0.025	0.030	0.04	0.05	0.06	0.08	1.0	15.0
		$C_{U_3O_8}$ = \$8/lb; C_N = \$0/g									
y =	0.00	84.052	131.63	(187.00)	(244.02)	340.03	432.49	523.21	699.62	4999.0	9413.4
	0.01	61.657	105.22	151.15	(209.13)	306.22	397.10	486.50	660.64	4916.2	9285.5
	0.025				(170.77)	266.17	352.41	438.40	607.46	4792.6	9094.2
		$C_{U_3O_8}$ = \$8/lb; C_N = \$60/g									
y =	0.00	84.023	131.59	180.60	(243.91)	340.29	432.84	523.64	700.19	5004.0	9418.1
	0.01	92.312	140.89	(191.34)	(253.52)	352.24	444.09	534.12	709.07	4970.4	9339.9
	0.025			(195.39)	(262.82)	365.94	457.76	547.27	720.55	4919.7	9222.4

Values in parentheses are from base case. Values at lower R are from pre-enrichment by gaseous diffusion. Values at higher R are from blending with natural uranium.

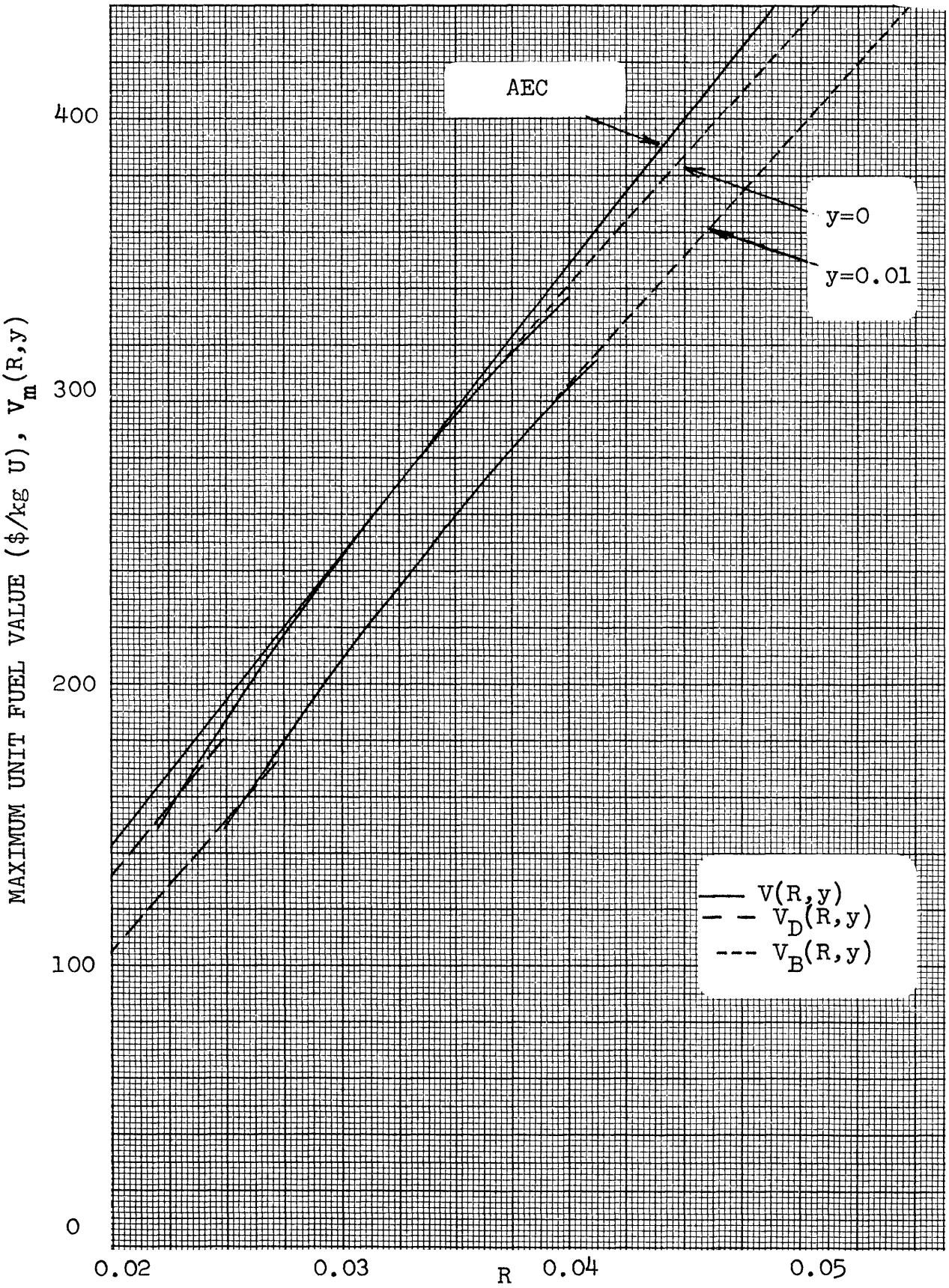


FIGURE IX.7 The Effect of R and y on the Maximum Unit Fuel Value - PWR, Spent U to HWR
$C_{U_3O_8}$ = \$8/lb C_N = \$0/g

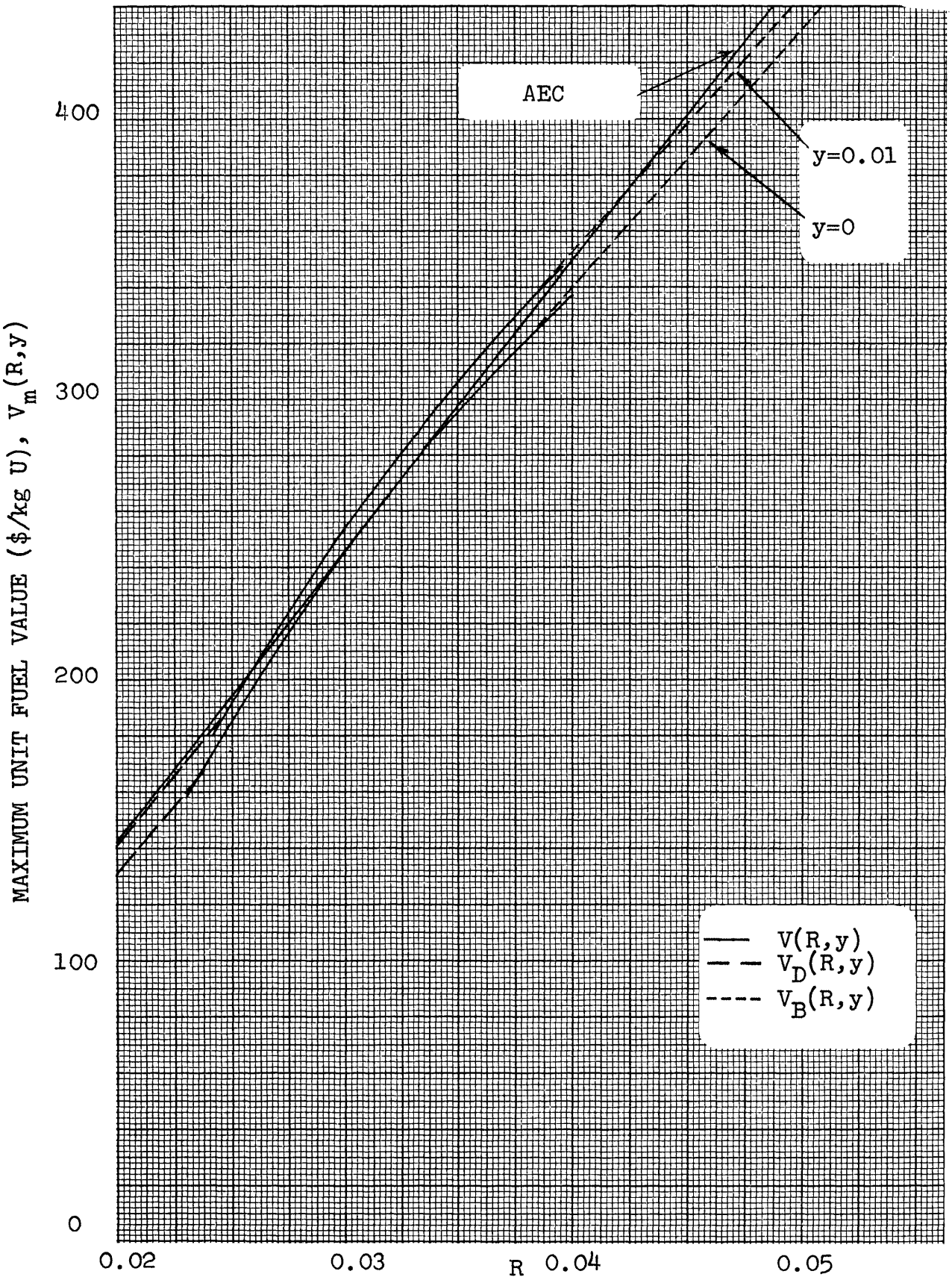


FIGURE IX.8 The Effect of R and y on the Maximum Unit Fuel Value - PWR, Spent Fuel to HWR
$C_{U_3O_8}$ = \$8/lb C_N = \$60/g

2. Comparison with Other Fuel Cycle Schemes

Curves comparing the maximum fuel values between the recycle to diffusion fuel cycle[1] and the spent U to HWR cycle are shown in Fig. IX.9. Since the modified modes of operation for each fuel cycle scheme are but modifications of the base case mode of operation, the trends resulting from the modified modes of operation will but reflect those found in the base case mode. Figure IX.9 demonstrates this for y=0.01 and a neptunium price of $0/g. It is of interest to note that the maximum fuel value curves parallel each other for the two fuel cycle schemes, the difference in value at any one enrichment being in the neighborhood of $60/kg U.

It was found that the maximum fuel value curves for the two fuel cycle schemes for y=0 almost coincided with each other. This was to be expected, however, since the base case curves for the two schemes nearly coincide and since the modified cases are but modifications of the base case.

I. U-236 Penalty for PWR with Spent Uranium Fed to HWR

In order to better characterize and describe the effect of U-236, a penalty for the presence of U-236 was defined in Eq. (VIII.1).

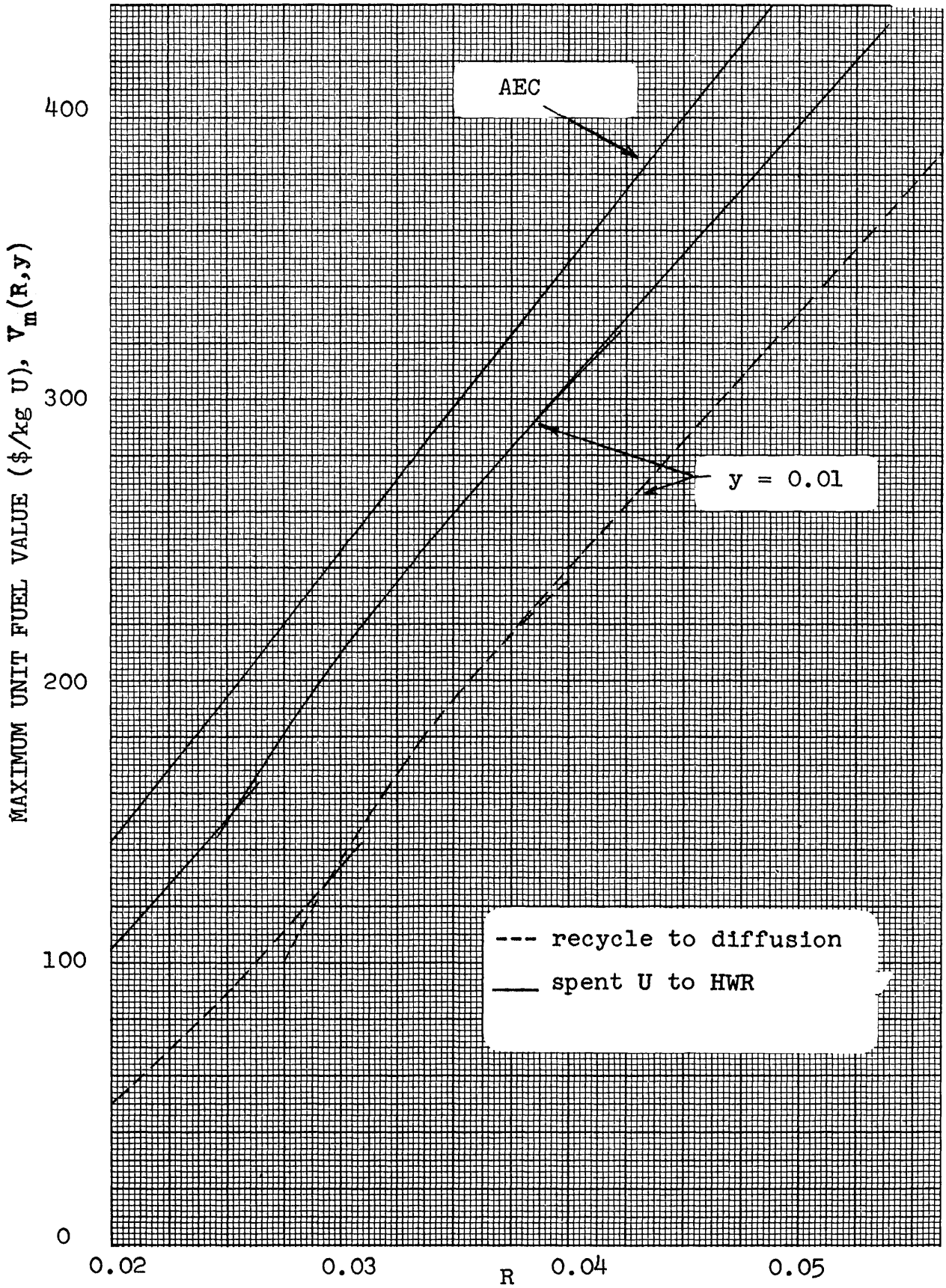


FIGURE IX.9 Comparison of the Maximum Unit Fuel Value for PWR with Spent Fuel Going to HWR and with Spent U Recycled through Diffusion Plant
$C_{U_3O_8}$ = \$8/lb C_N = \$0/g

The penalty values are shown graphically in Fig. IX.10. The dotted portions of the curves represent uncertainties in the penalty values in those regions. These uncertainties are small and are related to the fact that small differences are being taken graphically between large values, i.e., the maximum fuel values. A detailed explanation of the shape and uncertainties in the penalty curves may be found in section VIII. The important point to note in Fig. IX.10 is the magnitude of the U-236 penalty. When the neptunium price equals \$0/g, the penalty is approximately \$3.20/g for y=0.01, and when the neptunium price equals \$60/g, the penalty is in the neighborhood of -\$1.20/g U-236, the negative sign indicating that the presence of U-236 is no longer a penalty, but rather of economic advantage.

Using the two penalty values corresponding to the two neptunium prices, it is possible to calculate a neptunium price at which the U-236 penalty is zero. This price is known as the neptunium indifference value, C_N^o. At this price of neptunium it is a matter of indifference with regard to fuel cycle costs whether one purchases U-235 plus U-238 free of U-236 or the same amount of U-235 plus U-238 containing y kilograms of U-236 per kilogram of fuel. For y=0.01 the indifference value is \$43.95/g and for y=0.025 the indifference value is \$43.70/g. This indifference value is roughly the same as found in reference (1) for the recycle

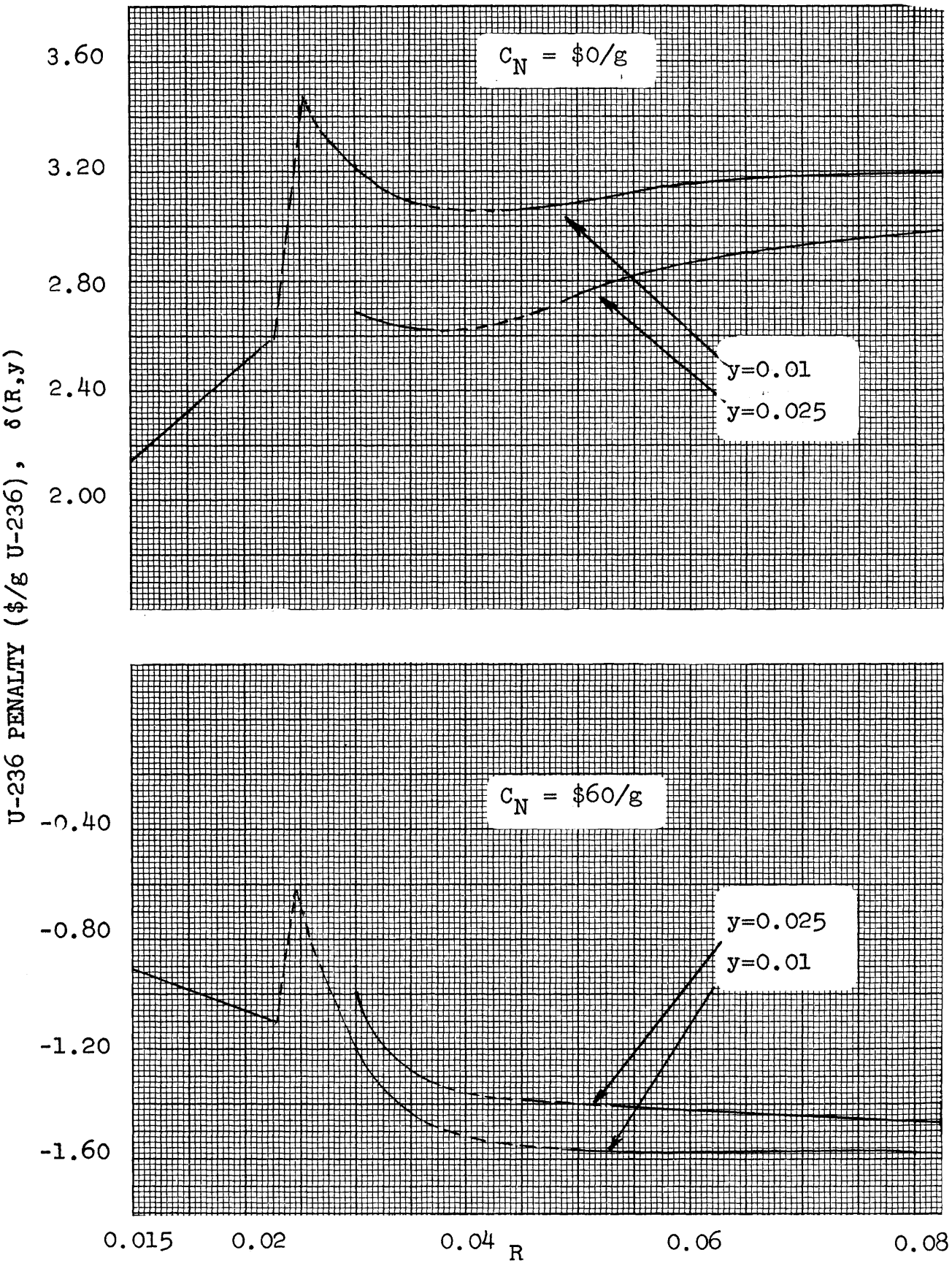


FIGURE IX.10 The Effect of R and y on the U-236 Penalty - PWR with Spent U Fed to HWR
$C_{U_3O_8}$ = \$8/lb

to diffusion cycle. On the other hand, the magnitude of the U-236 penalty when spent uranium is fed to an HWR is only about one-third the magnitude of the penalty when spent uranium is recycled through a diffusion plant. This is because of the buildup of U-236 in the reactor when uranium is recycled through the diffusion plant.

APPENDIX A

"AEC PRICE SCALE" FOR URANIUM AND FISSILE PLUTONIUM

The AEC price scale in effect in July 1967, for partially enriched uranium in the form of UF_6, is based on a price of \$8/lb U_3O_8 for natural uranium and a \$30/kg U price for separative work. This price scale is consistent with the equations

$$C_{AEC}(R) = C_\Delta \left[\frac{R-1}{R+1} \ln \frac{R}{R_T} + \frac{(R-R_T)(1-R_T)}{(R+1) R_T}\right] \qquad (A.1)$$

$$C_{NAT} = C_\Delta \left[\frac{R_{NAT} - 1}{R_{NAT} + 1} \ln \frac{R_{NAT}}{R_T} + \frac{(R_{NAT}-R_T)(1-R_T)}{(R_{NAT} + 1) R_T}\right] \qquad (A.2)$$

which are given in a slightly different, but equivalent, form in standard references such as reference (10). In these equations:

$C_{AEC}(R)$ is the price of uranium containing R weight ratio of U-235 to U-238 and no U-236, in the form of UF_6, \$/kg U

C_Δ is the unit cost of separative work, \$30/kg U

R_T is the optimum weight ratio of U-235 to U-238 in the diffusion plant tails, to be evaluated from Eq. (A.2)

C_{NAT} is the price of natural uranium in the form of UF_6, \$/kg U

R_{NAT} is the weight ratio of U-235 to U-238 in natural uranium, 0.00711/0.99289.

Values of C_{NAT} and R_T, corresponding to the natural U_3O_8 prices of \$6, \$8 and \$10/lb U_3O_8, are given in Table A.1.

TABLE A.1

Economic Variables Dependent on Price of Natural Uranium

Price of natural U_3O_8, $C_{U_3O_8}$, \$/lb	6	8	10
Cost of natural UF_6, C_{NAT}, \$/kg U	18.17	23.46	28.75
Optimum weight ratio U-235 to U-238 in diffusion plant tails, R_T	0.0028195	0.0025372	0.0023173
Credit for fissile plutonium, C_K, \$/g Pu	9.01	10.00	10.94

Throughout this work the term "AEC price scale" and the symbol $C_{AEC}(R)$ refer to the price for enriched UF_6 computed from Eqs. (A.1) and (A.2), using the appropriate price of natural U_3O_8 (\$6, \$8 or \$10/lb) and a separative work charge of \$30/kg U. It does not necessarily refer to the price charged by the AEC at any

particular time, although this price scale, when based on a \$8/lb U_3O_8 price, is indistinguishable from the AEC scale in effect in July, 1967.

The credit for fissile plutonium, C_K, at a given U_3O_8 price, is taken as 10/12 the price, in \$/g, of U-235 at 90% enrichment, as given by Eqs. (A.1) and (A.2) . Values of C_K corresponding to \$6, \$8, and \$10/lb U_3O_8 are given in Table A.1.

APPENDIX B

REFERENCE REACTOR CHARACTERISTICS

Table B.1 Reference HWOCR

POWER		
Fission Power (MWth)		3903
Net Plant Efficiency (%)		34.7
Net Plant Power Output (MWe)		1073
CALANDRIA		
Material		Stainless Steel
Height (inside) (FT)		20.0
Outside Diameter (FT)		25.0
Inside Diameter (FT)		24.83
Reflector Thickness		
Radial (FT)		1.0
Axial (FT)		1.0
PROCESS AND CALANDRIA TUBES		
Number		492
Lattice Arrangement		Square
Lattice Pitch (IN)		10.5
Gas Between Calandria and Process Tubes		CO_2
CO_2 Radial Gap (IN)		0.271
	Process Tube	Calandria Tube
Material	SAP-895	ZR-2
Thickness (IN)	0.116	0.052
Inside Diameter (IN)	4.32	5.094

FUEL ELEMENT

	Large Rods	Small Rods
Number Per Assembly	31	6
Hot Outside Diameter Excluding Fins (IN)	0.521	0.324
Hot Fuel Diameter (IN)	0.476	0.277
Cladding Thickness Excluding Fins (IN)	0.020	0.020
Number of Fins Per Rod	12	6 and 2
Fin Height, Nominal (IN)	0.080	0.060 and 0.128
Fin Thickness, Nominal (IN)	0.030	0.030 and 0.040

FUEL ASSEMBLY

Type of Fuel	UC
Type of Clad	SAP
Number of Assemblies Per Channel	5
Fuel Element Length (IN)	43.2
Average Total Active Fuel Length (FT)	17.3
Hot Fuel Assembly Outside Diameter (IN)	4.260
Cross Section Area of Fuel in Assembly (IN^2)	5.85
Total Core Loading (Metric Tons U)	123

COOLANT

Coolant	Santowax-OM
Coolant density, gm/cm^3 (with 10% High Boilers)	.837
Coolant Flow Rate, lb/hr	11×10^7

Coolant Temperature, ^{o}F	
Inlet	595
Outlet	750
Inlet Reactor Pressure, psia	284
Reactor Pressure Drop, psi	184
MODERATOR	
Moderator	D_2O
Moderator Temperature, ^{o}F	
Inlet	140
Outlet	200
Total Amount of D_2O in Calandria, lbs	588,000

TABLE B.2 Reference PWR - San Onofre Reactor
(Information from Reference(6))

POWER	
Total Heat Output (MW)	1346
Net Plant Efficiency (%)	31.9
Net Power Output (MWe)	430
GENERAL	
Total Core Area (Inside Core Baffle)(FT^2)	66.4
Equivalent Core Diameter (FT)	9.4
Maximum Diameter of Core (IN)	119.4
Core Length, between Fuel Ends (FT)	10
Lenght to Diameter Ratio of Core	1.09
Water to Uranium Ratio, Unit Cell	3.03
Fuel Weight, kg U	57,400
System Pressure, psi	2,100
Pressure Drop, psi	
Across Core	18.8
Across Vessel, including Nozzles	33
Core Power Density	
kw/liter of Core	71.6
kw/kg of U	23.4
FUEL ROD (COLD DIMENSIONS)	
Outside Diameter (IN)	0.422

Clad Material	Zircaloy
Clad Thickness, (IN)	0.0243
Diametral Gap (IN)	0.0055
Pellet Diameter (IN)	0.3835
Fuel Length (Pellets Only)(IN)	120
Pitch (IN)	0.556
Rod Array in Assembly	14 x 14
Rods per Assembly	180
Total Number Fuel Rods in Assemblies	28,260
Hydraulic Equivalent Diameter of Unit Cell (FT)	0.0426
Additional Water Gap Between Assemblies (IN)	0.019

COOLANT

Coolant	H_2O
Coolant Conditions	
Total Flow Rate, lbs/hr	76.9×10^6
Coolant Temperature, ^oF	
Inlet	552.8
Outlet	637.8

CORE

Total Core Volume (IN^3)	1,147,100
Weight Fraction of Material in Core	
Fuel	.312

Water	.581
Zircaloy 4	.088
Inconel	.004
Void	.009

APPENDIX C

INPUT DATA FOR CELL AND MOVE COMPUTER CODES FOR REFERENCE HWOCR

This appendix will contain some generalized comments concerning the methods used to arrive at the input data necessary for using CELL and MOVE codes for the reference HWOCR. The tabulated input data is listed in Tables C.1 and C.2; for symbol definitions refer to reference (8) for CELL code and reference (9) for MOVE code.

The initial concentrations of isotopes in the fuel is the atom fraction times the density times Avogadro's number divided by the molecular weight. The density of UC was taken to be 13.34 g/cc and the molecular weight 250.1. The reference case enrichment was 1.16 weight percent.

The concentration of cladding material was obtained in a similiar manner for SAP with a density of 2.77 g/cc and molecular weight 34.8. The concentration of the organic coolant was obtained from data given in reference (12). The number density for ZR-2 was obtained from reference (19). For D_2O a 99.75% purity and 190°F temperature gave a density of 1.0724 g/cc and molecular weight of 20.03.

The data given in section III and in Appendix B. were used to calculate all geometric parameters and

volume fractions. In cases where more than one geometric size occurs in the reactor (e.g. fuel pin diameter) a weighted average was used. The volume of the clad was assumed to include the clad, the fins, the end plugs, and the end plates. The end plates are included even though they are ZR-4 because the volume contribution of the end plates is small and the properties of ZR-4 are not significantly different from those of SAP.

The disadvantage factors for the extra region materials as described in reference (8) were obtained by Olson[13] using the THERMOS code.

Table C.1 gives a brief definition of the major cross section symbols and the material identification. In Table C.2 the cross sectional data is listed with appropriate references.

Those resonance integrals identified by reference (8) were obtained using the hand calculational method described therein.

The effective fuel temperature was obtained from reference (7). The effective moderator temperature was the homogenized slowing down power weighted average of the organic and heavy water temperatures.[8]

The Fermi Age used was the volume weighted Fermi Age for the organic and heavy water; the diffusion coefficient was obtained in a similiar manner. The Fermi Age of the terphenyl was obtained by interpolating

TABLE C.1

CROSS SECTION SYMBOLS

SAO(K)	Absorption Cross Section, 2200 m/sec,
STR(K)	Effective Thermal Scattering Cross Section, $(1-\bar{\mu})\sigma_s$
ESSR(K)	Slowing Down Power, $\xi\sigma_s^{RES}$
RINT(K)	Resonance Integral

K = 1	UC
2	SAP
3	Terphenyl
4	SAP
5	CO_2
6	ZR-2
7	D_2O
8	Unused

the carbon-hydrogen ratio (14) with the result being 48.6 cm^2.

In calculating the geometric buckling, the effective axial reflector savings was taken to be zero due to parasitic absorption in the axial reflector region. (7) The radial reflector thickness is one foot and is assumed equal to the radial reflector savings. Hence

$$B_g^2 = \left(\frac{2.405}{R + \delta R}\right)^2 + \left(\frac{\pi}{H}\right)^2$$

From this and the Fermi Age, the fast non-leakage probability was calculated

$$P_{1IN} = \frac{1}{1 + B_g^2 \tau}$$

The fast fission factor was calculated by the method of Spinrad, Fleishman, and Soodak as described in reference (15). The fast effect cross sections for U^{238} were obtained from reference (16) and for C from reference (17). The collision probabilities were obtained from reference (14) and a correction for the Dancoff factor was applied.

The thermal cross section data, the lethargy increments, the resonance cross section data, and the Wigner-Wilkins startup data which were used are

described in reference (8).

In the MOVE code, ten radial mesh points were chosen in such a manner that the core was divided into two equal volume zones. It should be pointed out that this was done to parallel the methods used in reference (4) and that the two radial zones do not refer to two zones of different initial enrichment.

The continuous bidirectional fuel management scheme with axial velocity specified was used but only after discontinuous bidirectional fuel management was determined to give less representative results. The relative axial velocities in the reference reactor were adjusted to reflect the relative residence times predicted by reference (4) and to approximate the power distribution predicted by reference (4).

TABLE C.2

REFERENCE CELL INPUT DATA

Symbol		Reference
ANIN(5)	= 3.774 E-04	
ANIN(6)	= 0	
ANIN(7)	= 0	
ANIN(8)	= 0.03175	
ANIN(9)	= 0	
ANIN(10)	= 0	
ANIN(11)	= 0	
ANIN(12)	= 0	
ANIN(13)	= 0	
ACLD	= 0.04794	
ACOL	= 0.0791	
RAD	= 0.5690	
R1	= 0.9017	
R2	= 0.6266	
TC	= 0.0508	
ZLAT	= 1.0	
VFF	= 0.04693	
VFVD	= 0.00300	
VFCLD	= 0.01845	
VFCOL	= 0.05396	
VEX	= 0.87767	
VEM(1)	= 0.01537	
VEM(2)	= 0.03905	
VEM(3)	= 0.00800	

TABLE C.2

(Continued)

VEM(4)	= 0.93758	
VEM(5)	= 0	
ANN(1)	= 0.04794	
ANN(2)	= 0	
ANN(3)	= 0.04326	
ANN(4)	= 0.03225	
ANN(5)	= 0	
DIFAC(1)	= 1.432	
DIFAC(2)	= 1.510	
DIFAC(3)	= 1.541	
DIFAC(4)	= 2.046	
DIFAC(5)	= 0	
SAO(1)	= 0	(8)
SAO(2)	= 0.241	(14)
SAO(3)	= 0.102	(12)
SAO(4)	= 0.241	(14)
SAO(5)	= 0	(8)
SAO(6)	= 0.21	(18)
SAO(7)	= 0.00267	(14)
SAO(8)	= 0	
STR(1)	= 12.9	(19)
STR(2)	= 1.37	(14)
STR(3)	= 9.84	(12)
STR(4)	= 1.37	(14)
STR(5)	= 0	(8)

TABLE C.2

(Continued)

STR(6)	= 6.06	(19)(14)
STR(7)	= 11.6	(18)
STR(8)	= 0	
SCPFA	= 4.7	(19)
SSRCL	= 1.4	(19)
SSRCO	= 10.11	(12)
ESSR(1)	= 0.8324	(19)
ESSR(2)	= 0.1011	(19)
ESSR(3)	= 9.2	(12)
ESSR(4)	= 0.1011	(19)
ESSR(5)	= 0	(8)
ESSR(6)	= 0.1328	(19)
ESSR(7)	= 5.38	(19)
ESSR(8)	= 0	
RINT(1)	= 0	(8)
RINT(2)	= 0.191	(20)
RINT(3)	= 0.0502	(8)
RINT(4)	= 0.191	(20)
RINT(5)	= 0	(8)
RINT(6)	= 1.51	(20)
RINT(7)	= 0.00132	(8)
RINT(8)	= 0	
RIUFF	= 181.0	
RIPFP	= 264.0	
TMOD	= 137.5	

TABLE C.2

(Continued)

TEFF	= 811.
TAU	= 120.3
P1IN	= 0.9913
POWERD	= 14.8
PDNLIM	= 41.9
ENNFIS(1)	= 199.1
ENNFIS(2)	= 199.1
ENNFIS(3)	= 199.1
ENNFIS(4)	= 199.1
SFAC(1)	= 1.0
SFAC(2)	= 1.0
XEADJ	= 1.0
SMADJ	= 1.0
FPFCTR	= 1.0
ZETA	= 0.0007
EVCUT	= 0.414
B22	= 7.33 E-05
EPSI	= 1.0191
RI8CHK	= 0
IL	= 49
NRES	= 68
NUMPOZ	= 58
NUMSPA	= 4
NWILK	= 1
NPOILK	= 0

TABLE C.2

(Continued)

NPT = 3

NWT = 5

ISKIP = 0

INPUT = 1

IPRNT = 1

IPRT1 = 0

IPRT2 = 0

IPRWLK = 0

Thermal Cross Section Data	(8)
Lethargy Increments	(8)
Resonance Cross Section Data	(8)
Wigner-Wilkins Startup Data	(8)

TABLE C.3

REFERENCE MOVE INPUT DATA

R(1)	= 35.14
R(2)	= 70.28
R(3)	= 105.42
R(4)	= 140.55
R(5)	= 175.69
R(6)	= 210.83
R(7)	= 245.97
R(8)	= 279.96
R(9)	= 313.94
R(10)	= 347.93
H	= 548.6
δR	= 30.5
δH	= 0
ZSYM	= 0
DBSQU	= 6.3 E-05
PFAST	= 0.9913
PDENAV	= 14.8
RMAX	= 2.83
ERROR	= 0.005
DELCRT	= 0.0005
DELTD	= 0
CRIT	= 1.0
NZONE(1)	= 7
NZONE(2)	= 3
NZONE(3)	= 0

TABLE C.3

(Continued)

NZONE(4) = 0

NZONE(5) = 0

LOCPRP(1) = 1

LOCPRP(2) = 0

LOCPRP(3) = 0

LOCPRP(4) = 0

LOCPRP(5) = 0

IPROP(1) = 1

IPROP(2) = 1

IPROP(3) = 0

IPROP(4) = 0

IPROP(5) = 0

IRL = 10

JZL = 15

IZONE = 2

NLOAD = 1

NOT = 2

NRT = 5

IMUV = 3

IPOIS = 1

NPOISR = 0

NPOISR = 0

NSTEP = 0

ISSCNT = 1

IBATCH = 0

TABLE C.3

(Continued)

IGNOR = 2

ITRATE = 20

IPRT1 = 0

IPRT2 = 0

IPSPPR = 0

IPSGMW = 0

IPOWD = 0

INORMP = 0

IABSP = 0

ITHET = 0

ICSTRD = 0

THETA1 = 0.011

THETA2 = 0.013

DAMP1 = 0.25

EFF = 0

ERROR = 0.005

DELCNV = 0.0004

LPMX = 0

NEXT = 0

FCTR(1) = 1.000

FCTR(2) = 0.992

FCTR(3) = 0.983

FCTR(4) = 0.975

TABLE C.3

(Continued)

FCTR(5) = 0.966

FCTR(6) = 0.958

FCTR(7) = 0.949

FCTR(8) = 1.235

FCTR(9) = 0.923

FCTR(10) = 0.509

APPENDIX D

DIFFUSION PLANT EQUATIONS

In the modified case with pre-enrichment by gaseous diffusion, it has been assumed that the diffusion plant is operated in such a manner that at each point where two streams are mixed the U-235 to U-238 weight ratio of the two streams is the same. De la Garza, Garrett, and Murphy,(11) call a diffusion cascade operated in this manner a "matched R cascade"; they have also shown that the distribution of U-236 between product and waste in a matched R cascade is given by

$$\frac{y_D F}{(R_D)^{1/3}} + \frac{y_T F_T}{(R_T)^{1/3}} = \frac{y F_D}{(1+L_C)(R)^{1/3}} \qquad (D.1)$$

R_T is the optimum tails weight ratio which is determined from the cost of natural uranium and the cost of separative work.

There are also three mass balance relations for the diffusion plant which are given below.

$$F + F_T = \frac{F_D}{1 + L_C} \qquad \text{(Total U)} \qquad (D.2)$$

$$y_D F + y_T F_T = \frac{y F_D}{1 + L_C} \qquad \text{(U-236)} \qquad (D.3)$$

$$\frac{R_D(1-y_D)}{1+R_D} F + \frac{R_T(1-y_T)}{1+R_T} F_T = \frac{R(1-y)}{1+R} F_D \qquad \text{(U-235)} \qquad (D.4)$$

With Equations D.1 through D.4 and the fact that F is a function of R_D and y_D as is determined from the

base case results, the value of y_D can be determined for a given R and y and an assumed R_D. The specification of R, y, and R_D allows all the steady state characteristics of the diffusion plant to be determined.

The separative work expended per day, on the average, in a matched R cascade as described previously is

$$\Delta_D = F\left[\frac{2R_D(1-y_D)}{1+R_D} + 4y_D - 1\right] \ell nR_D + F_T\left[\frac{2R_T(1-y_T)}{1+R_T} + 4y_T - 1\right] \ell nR_T - \frac{F_D}{(1+L_C)}\left[\frac{2R(1-y)}{1+R} + 4y - 1\right] \ell nR \quad (D.5)$$

With the known cost of separative work, C_Δ, the cost of pre-enriching feed in the diffusion plant is $\Delta_D C_\Delta$, \$/day.

APPENDIX E

E.1. Fuel Values for Modified Case with Pre-Enrichment by Gaseous Diffusion ($/kg U). HWR.

Case 1 $C_{U_3O_8}$ = $10/lb C_N = $0/g

R =	0.004	0.006	0.008	0.010	0.011	0.012	0.013	0.014
y = 0								
$V_D(R,y)$	0.091	13.28	29.90	48.36	58.04	67.96		
R_D	0.01280	0.0128	0.0128	0.0128	0.0128	0.0128		
y_D	0	0	0	0	0	0		
F_D	793.967	363.5	236.0	174.93	154.9	139.0		
y_T	0	0	0	0	0	0		
F_T	663.4	234.2	107.1	46.23	26.29	10.48		
y = 0.005								
$V_D(R,y)$	-2.24	9.21	24.65	42.21	51.52	61.09	70.87	
R_D	0.0144	0.0140	0.0138	0.0136	0.0136	0.0136	0.0136	
y_D	0.01286	0.00944	0.00756	0.00632	0.00588	0.00550	0.00517	
F_D	917.1	407.4	259.5	189.9	167.3	149.5	135.1	
y_T	0.00370	0.00291	0.00244	0.00212	0.00200	0.00189	0.00179	
F_T	784.71	276.5	129.6	59.79	38.0	20.87	7.06	
y = 0.010								
$V_D(R,y)$	-3.74	6.47	20.78	37.37	46.26	55.45	64.88	
R_D	0.0150	0.0150	0.0148	0.0144	0.0144	0.0142	0.0140	
y_D	0.02632	0.0198	0.0159	0.0132	0.0122	0.0113	0.0105	
F_D	1019.9	443.4	280.1	203.7	178.9	159.51	143.8	
y_T	0.00742	0.00587	0.00493	0.00429	0.00404	0.00381	0.00361	
F_T	878.2	311.2	150.5	72.9	49.47	28.98	12.06	

Fuel Values for Modified Case with Pre-Enrichment by Gaseous Diffusion ($/kg U), HWR (Continued)

$C_{U_3O_8}$ = \$10/lb C_N = \$0/g

R =	0.004	0.006	0.008	0.010	0.011	0.012	0.013	0.014
y = 0.020								
$V_D(R,y)$					38.64	47.31	56.20	65.29
R_D					0.0150	0.0150	0.0150	0.0150
y_D					0.0252	0.0236	0.0223	0.0210
F_D					199.2	176.5	158.4	143.6
y_T					0.0081	0.0077	0.0073	0.0070
F_T					61.3	40.7	24.3	11.0

Case 3 $C_{U_3O_8}$ = \$10/lb C_N = \$60/g

R =	0.004	0.006	0.008	0.010	0.011	0.012	0.013	0.014
y = 0								
$V_D(R,y)$	0.087	13.27	29.88	48.34	58.02	67.93		
R_D	0.0130	0.0130	0.0130	0.0130	0.0130	0.0130		
y_D	0	0	0	0	0	0		
F_D	793.21	363.15	235.81	174.77	154.79	138.94		
y_T	0	0	0	0	0	0		
F_T	665.15	236.38	109.42	48.56	28.64	12.84		
y = 0.005								
$V_D(R,y)$	1.50	15.70	32.99	51.94	61.81	71.89	82.13	
R_D	0.0138	0.0136	0.0136	0.0138	0.0138	0.0138	0.0136	
y_D	0.0124	0.00923	0.00742	0.00639	0.00595	0.00556	0.00517	
F_D	919.03	407.98	259.76	189.84	167.27	149.47	135.15	
y_T	0.00369	0.00291	0.00244	0.00213	0.00200	0.00189	0.00179	
F_T	779.67	272.41	127.49	61.98	40.19	23.06	7.06	

Fuel Values for Modified Case with Pre-Enrichment by Gaseous Diffusion ($/kg U), HWR

(Continued)

$C_{U_3O_8}$ = \$10/lb C_N = \$60/g

R =	0.004	0.006	0.008	0.010	0.011	0.012	0.013	0.014
y = 0.010								
$V_D(R,y)$	2.48	17.48	32.28	54.64	64.69	74.94	85.33	
R_D	0.0146	0.0144	0.0140	0.0138	0.0136	0.0136	0.0136	
y_D	0.0257	0.0192	0.0152	0.0127	0.0117	0.0110	0.0103	
F_D	1021.74	444.80	281.09	204.16	179.47	159.86	144.06	
y_T	0.00741	0.00585	0.00491	0.00427	0.00401	0.00379	0.00360	
F_T	875.37	305.90	142.38	66.46	40.63	22.24	7.50	
y = 0.020								
$V_D(R,y)$				58.74	69.28	79.87	90.51	101.22
R_D				0.0146	0.0146	0.0146	0.0146	0.0144
y_D				0.0265	0.0247	0.0232	0.0218	0.0204
F_D				229.10	199.65	176.90	158.78	144.14
y_T				0.00864	0.00814	0.00770	0.00732	0.00696
F_T				83.92	57.15	36.52	20.14	4.64

Case 5

$C_{U_3O_8}$ = \$8/lb C_N = \$0/g

R =	0.004	0.006	0.008	0.010	0.011	0.012	0.013	0.014
y = 0								
$V_D(R,y)$	-1.60	9.41	23.85	40.15	48.76	57.60	66.62	
R_D	0.0132	0.0132	0.0132	0.0132	0.0132	0.0132	0.0132	
y_D	0	0	0	0	0	0	0	
F_D	893.6	378.2	240.2	176.2	155.5	139.2	126.1	
y_T	0	0	0	0	0	0	0	
F_T	767.6	253.8	116.2	52.3	31.7	15.5	2.3	

Fuel Values for Modified Case with Pre-Enrichment by Gaseous Diffusion ($/kg U), HWR

(Continued)

$C_{U_3O_8}$ = $8/lb C_N = $0/g

R =	0.004	0.006	0.008	0.010	0.011	0.012	0.013	0.014
y = 0.005								
$V_D(R,y)$	-3.44	5.92	19.24	34.70	42.95	51.47	60.20	
R_D	0.0150	0.0144	0.0140	0.0140	0.0140	0.0138	0.0138	
y_D	0.0132	0.0096	0.0076	0.0065	0.0060	0.0056	0.0052	
F_D	1034.0	425.0	264.8	191.7	168.3	150.0	135.3	
y_T	0.0039	0.0031	0.0026	0.0022	0.0021	0.0020	0.0019	
F_T	907.5	298.4	137.1	66.0	43.4	23.6	9.5	
y = 0.010								
$V_D(R,y)$	-4.64	3.57	15.89	30.44	38.29	46.44	54.83	63.44
R_D	0.0154	0.0152	0.0152	0.0150	0.0148	0.0146	0.0146	0.0144
y_D	0.0266	0.0199	0.0162	0.0136	0.0125	0.0116	0.0109	0.0102
F_D	1148.1	462.6	285.7	205.6	180.2	160.2	144.1	130.9
y_T	0.0078	0.0062	0.0052	0.0045	0.0043	0.0040	0.0038	0.0036
F_T	1010.7	332.7	160.3	81.2	54.9	33.9	18.7	4.3
y = 0.020								
$V_D(R,y)$					31.42	39.10	47.02	55.14
R_D					0.0154	0.0152	0.0152	0.0152
y_D					0.0258	0.0239	0.0225	0.0213
F_D					200.4	177.3	158.8	143.8
v_T					0.0086	0.0081	0.0077	0.0074
F_T					67.0	43.7	26.9	1325.9

Fuel Values for Modified Case with Pre-Enrichment by Gaseous Diffusion ($/kg U), HWR (Continued)

Case 7 $C_{U_3O_8}$ = $8/lb C_N = $60/g

R =	0.004	0.006	0.008	0.010	0.011	0.012	0.013	0.014
y = 0								
$V_D(R,y)$	-1.60	9.39	23.83	40.12	48.73	57.56	66.58	
R_D	0.0132	0.0132	0.0132	0.0132	0.0132	0.0132	0.0132	
y_D	0	0	0	0	0	0	0	
F_D	893.6	378.2	240.2	176.2	155.5	139.2	126.1	
y_T	0	0	0	0	0	0	0	
F_T	767.6	253.8	116.2	52.3	31.7	15.5	2.3	
y = 0.005								
$V_D(R,y)$	-0.17	12.07	27.36	44.28	53.14	62.20	71.42	
R_D	0.0140	0.0138	0.0138	0.0140	0.0140	0.0140	0.0140	
y_D	0.0125	0.0093	0.0075	0.0065	0.0060	0.0056	0.0053	
F_D	1035.4	425.0	264.3	191.7	168.3	150.0	135.3	
y_T	0.0039	0.0031	0.0025	0.0022	0.0021	0.0020	0.0019	
F_T	898.3	291.9	134.9	66.0	43.4	25.8	11.6	
y = 0.010								
$V_D(R,y)$	0.80	14.01	29.91	47.33	56.40	65.66	75.07	
R_D	0.0146	0.0146	0.0142	0.0140	0.0140	0.0138	0.0138	
y_D	0.0256	0.0193	0.0154	0.0129	0.0120	0.0111	0.0105	
F_D	1152.2	463.7	286.6	206.0	180.3	160.3	144.1	
y_T	0.0078	0.0062	0.0052	0.0045	0.0042	0.0040	0.0038	
F_T	1005.8	327.2	150.3	70.7	46.4	25.1	10.0	

Fuel Values for Modified Case with Pre-Enrichment by Gaseous Diffusion ($/kg U), HWR

(Continued)

$C_{U_3O_8}$ = \$8/lb C_N = \$60/g

R =	0.004	0.006	0.008	0.010	0.011	0.012	0.013	0.014
y = 0.020								
$V_D(R,y)$				52.02	61.66	71.33	81.06	90.85
R_D				0.0146	0.0146	0.0146	0.0146	0.0146
y_D				0.0265	0.0247	0.0232	0.0218	0.0206
F_D				231.5	201.1	177.7	159.2	144.1
y_T				0.0091	0.0085	0.0081	0.0077	0.0073
F_T				86.4	58.7	37.4	20.6	7.0

Case 8

$C_{U_3O_8}$ = \$8/lg C_N = \$100/g

R =	0.004	0.006	0.008	0.010	0.011	0.012	0.013	0.014
y = 0.0								
$V_D(R,y)$	-1.068	9.38	23.81	40.10	48.70	57.53	66.55	
R_D	0.0134	0.0134	0.0134	0.0134	0.0134	0.0134	0.0134	
y_D	0	0	0	0	0	0	0	
F_D	893.8	378.3	240.2	176.2	155.5	139.2	126.0	
y_T	0	0	0	0	0	0	0	
F_T	770.0	256.0	118.4	54.59	33.9	17.72	4.58	
y = 0.005								
$V_D(R,y)$	2.03	16.19	32.78	50.67	59.93	69.35	78.90	
R_D	0.0136	0.0138	0.0138	0.0138	0.0140	0.0140	0.0140	
y_D	0.0122	0.00929	0.00753	0.00638	0.00600	0.00562	0.00529	
F_D	1036.8	425.0	264.7	191.6	168.3	150.0	135.2	
y_T	0.00387	0.00306	0.00257	0.00223	0.00210	0.00199	0.00189	
F_T	894.7	291.8	134.88	63.82	43.42	25.77	11.60	

Fuel Values for Modified Case with Pre-Enrichment by Gaseous Diffusion ($/kg U), HWR

(Continued)

$C_{U_3O_8}$ = \$8/lb C_N = \$100/g

R =	0.004	0.006	0.008	0.010	0.011	0.012	0.013	0.014
y = 0.010								
$V_D(R,y)$	4.45	21.03	39.42	58.75	68.62	78.60	88.65	
R_D	0.0144	0.0142	0.0136	0.0136	0.0136	0.0136	0.0136	
y_D	0.0253	0.0189	0.0148	0.0126	0.0117	0.0109	0.0103	
F_D	1152.9	464.4	287.3	206.2	180.5	160.38	144.2	
y_T	0.00778	0.00614	0.00514	0.00448	0.00421	0.00398	0.00378	
F_T	1003.9	323.2	143.5	66.06	41.70	22.77	7.66	
y = 0.020								
$V_D(R,y)$				70.82	81.92	92.93	103.90	
R_D				0.0144	0.0144	0.0144	0.0144	
y_D				0.0262	0.0244	0.0229	0.0216	
F_D				231.6	201.2	177.8	159.32	
y_T				0.00905	0.00853	0.00807	0.00767	
F_T				84.08	56.36	35.11	18.32	

Case 9

$C_{U_3O_8}$ = \$6/lb C_N = \$0/g

R =	0.004	0.006	0.008	0.010	0.011	0.012	0.013	0.014
y = 0.0								
$V_D(R,y)$	-3.09	5.560	17.74	31.75	39.21	46.91	54.79	
R_D	0.0136	0.0136	0.0136	0.0136	0.0136	0.0136	0.0136	
y_D	0	0	0	0	0	0	0	
F_D	1079.8	401.6	247.05	178.59	156.92	139.96	126.34	
y_T	0	0	0	0	0	0	0	
F_T	957.6	281.3	127.29	59.03	37.42	20.52	6.940	

Fuel Values for Modified Case with Pre-Enrichment by Gaseous Diffusion ($/kg U), HWR

(Continued)

$C_{U_3O_8}$ = $6/lb C_N = $0/g

R =	0.004	0.006	0.008	0.010	0.011	0.012	0.013	0.014
y = 0.005								
$V_D(R,y)$	-4.42	2.78	13.81	27.02	34.15	41.55	49.17	
R_D	0.0156	0.0148	0.0146	0.0144	0.0144	0.0142	0.0142	
y_D	0.01343	0.00974	0.00783	0.00658	0.00613	0.00568	0.00534	
F_D	1248.8	451.96	272.83	194.58	170.11	150.93	135.72	
y_T	0.00412	0.00325	0.00273	0.00237	0.00223	0.00211	0.00200	
F_T	1127.7	329.36	151.27	72.99	49.25	28.74	14.08	
y = 0.010								
$V_D(R,y)$	-5.30	0.78	10.99	23.40	30.16	37.20	44.50	51.99
R_D	0.0162	0.0156	0.0158	0.0156	0.0154	0.0152	0.0152	0.0150
y_D	0.0274	0.0201	0.0165	0.0139	0.0128	0.0119	0.0112	0.0105
F_D	1372.8	491.04	293.38	208.44	181.93	161.26	144.69	131.15
y_T	0.00825	0.00653	0.00551	0.00480	0.00452	0.00427	0.00406	0.00386
F_T	1244.2	365.29	174.07	89.94	62.58	40.94	25.22	10.55
y = 0.020								
$V_D(R,y)$					24.40	30.66	37.54	44.64
R_D					0.0158	0.0156	0.0156	0.0156
y_D					0.0261	0.0243	0.0229	0.0216
F_D					201.99	178.32	159.28	143.84
y_T					0.00911	0.00861	0.00819	0.00781
F_T					7.305	49.05	31.60	17.53

Fuel Values for Modified Case with Pre-Enrichment by Gaseous Diffusion ($/kg U), HWR (Continued)

Case 11 $C_{U_3O_8}$ = \$6/lb C_N = \$60/g

R =	0.004	0.006	0.008	0.010	0.011	0.012	0.013	0.014
y = 0.0								
$V_D(R,y)$	-3.10	5.59	17.72	31.72	39.18	46.87	54.73	
R_D	0.0138	0.0138	0.0138	0.0138	0.0138	0.0138	0.0138	
y_D	0	0	0	0	0	0	0	
F_D	1081.2	402.11	247.3	178.8	157.1	140.1	126.4	
y_T	0	0	0	0	0	0	0	
F_T	960.9	283.8	129.59	61.25	39.62	22.60	9.09	
y = 0.005								
$V_D(R,y)$	-1.77	8.42	21.61	36.40	44.18	52.16	60.31	
R_D	0.0142	0.0142	0.0142	0.0142	0.0142	0.0142	0.0142	
y_D	0.0125	0.00943	0.00767	0.00651	0.00606	0.00568	0.00534	
F_D	1250.2	451.16	272.4	194.4	169.9	150.9	135.7	
y_T	0.00411	0.00324	0.00272	0.00237	0.00223	0.00211	0.00200	
F_T	1114.8	322.5	146.8	70.83	47.12	28.74	14.08	
y = 0.010								
$V_D(R,y)$	-0.87	10.47	24.40	39.77	47.81	56.03	64.40	
R_D	0.0148	0.0148	0.0146	0.0144	0.0142	0.0142	0.0142	
y_D	0.0256	0.0193	0.0156	0.0131	0.0121	0.0113	0.0106	
F_D	1391.2	492.4	294.6	208.7	181.9	161.0	144.4	
y_T	0.00824	0.00652	0.00548	0.00476	0.00448	0.00424	0.00403	
F_T	1246.6	358.2	163.0	78.12	50.30	30.58	14.94	

Fuel Values for Modified Case with Pre-Enrichment by Gaseous Diffusion ($/kg U), HWR (Continued)

$C_{U_3O_8}$ = \$6/lb C_N = \$60/g

R =	0.004	0.006	0.008	0.010	0.011	0.012	0.013	0.014
y = 0.020								
$V_D(R,y)$				45.05	53.72	62.44	71.21	80.04
R_D				0.0148	0.0148	0.0148	0.0148	0.0148
y_D				0.0267	0.0249	0.0233	0.0220	0.0208
F_D				234.8	203.0	178.8	159.7	144.2
y_T				0.00961	0.00905	0.00857	0.00814	0.00776
F_T				92.15	62.98	40.81	23.39	9.38

APPENDIX F

F.1 Fuel Values for Modified Case with Blending with Natural Uranium ($/kg U). HWR.

Case 1 $C_{U_3O_8}$ = \$10/lg C_N = \$0/g

	R = 0.016	0.018	0.020
y = 0			
$V_B(R,y)$	115.53	134.96	154.31
	0.2487	0.3879	0.4813
R_B	0.0137	0.0137	0.0137
y_B	0	0	0
F_B	88.01	71.81	60.79
y = 0.005			
$V_B(R,y)$	108.19	127.48	146.70
	0.2038	0.3561	0.4574
R_B	0.0141	0.0141	0.0140
y_B	0.00398	0.00321	0.0027
F_B	95.04	76.62	64.3
y = 0.010			
$V_B(R,y)$	101.57	120.49	139.47
	0.1309	0.3233	0.4364
R_B	0.0148	0.0144	0.0143
y_B	0.00869	0.00676	0.00563
F_B	103.9	81.74	67.84
y = 0.020			
$V_B(R,y)$	91.73	109.70	127.61
	0.00924	0.1679	0.2925
R_B	0.01591	0.01613	0.01615
y_B	0.01981	0.01664	0.01414
F_B	119.2	95.65	79.50

Case 3 $C_{U_3O_8}$ = \$10/lb C_N = \$60/g

	R = 0.016	0.018	0.020
y = 0			
$V_B(R,y)$	115.65	135.10	154.48
	0.2287	0.36893	0.46651
R_B	0.01396	0.01397	0.01397
y_B	0	0	0
F_B	89.036	72.795	61.560

Fuel Value for Modified Case with Blending with Natural Uranium ($/kg U), HWR (Continued)

	R = 0.016	0.018	0.020
y = 0.005			
$V_B(R,y)$	120.77	140.27	159.68
	0.17587	0.32812	0.43349
R_B	0.01442	0.01440	0.01437
y_B	0.00142	0.00335	0.00283
F_B	96.542	78.10	65.56
y = 0.010			
$V_B(R,y)$	124.54	144.22	163.74
	0.15499	0.3193	0.4244
R_B	0.01460	0.01449	0.01447
y_B	0.00845	0.00680	0.00575
F_B	102.67	81.96	68.49
y = 0.020			
$V_B(R,y)$	131.22	150.97	170.36
	0.0452	0.1919	0.3205
R_B	0.01558	0.01586	0.01579
y_B	0.01909	0.01612	0.01358
F_B	117.69	94.77	78.49

Case 5 $C_{U_3O_8}$ = \$8/lb C_N = \$0/g

	R = 0.016	0.018	0.020
y = 0			
$V_B(R,y)$	100.87	118.19	135.45
	0.169	0.321	0.423
R_B	0.0145	0.0145	0.0145
y_B	0	0	0
F_B	92.0	75.1	63.7
y = 0.005			
$V_B(R,y)$	94.18	111.37	128.52
	0.160	0.316	0.425
R_B	0.0146	0.0145	0.0145
y_B	0.0042	0.0034	0.0029
F_B	93.4	78.7	66.0

Fuel Value for Modified Case with Blending with Natural Uranium ($/kg U), HWR (Continued)

	R = 0.016	0.018	0.020
y = 0.010			
$V_B(R,y)$	88.34	105.10	121.98
	0.035	0.272	0.402
R_B	0.0157	0.0150	0.0148
y_B	0.0097	0.0073	0.0060
F_B	108.6	84.5	69.7
y = 0.020			
$V_B(R,y)$		95.67	111.78
		0.144	0.265
R_B		0.0164	0.0165
y_B		0.0171	0.0147
F_B		96.4	80.3

Case 7 $C_{U_3O_8}$ = $8/lb C_N = $60/g

	R = 0.016	0.018	0.020
y = 0			
$V_B(R,y)$	100.99	118.34	135.63
	0.141	0.230	0.404
R_B	0.0147	0.0147	0.0148
y_B	0	0	0
F_B	93.3	76.2	64.6
y = 0.005			
$V_B(R,y)$	106.77	124.21	141.53
	0.144	0.304	0.409
R_B	0.0147	0.0147	0.0147
y_B	0.0043	0.0035	0.0030
F_B	98.2	79.3	66.8
y = 0.010			
$V_B(R,y)$	111.13	128.73	146.20
	0.103	0.288	0.402
R_B	0.0151	0.0148	0.0148
y_B	0.0090	0.0071	0.0060
F_B	105.4	83.6	69.7

Fuel Value for Modified Case with Blending with Natural Uranium ($/kg U), HWR (Continued)

	R = 0.016	0.018	0.020
y = 0.020			
$V_B(R,y)$	118.81	136.78	154.34
	0.025	0.168	0.277
R_B	0.0158	0.0161	0.0164
y_B	0.0195	0.0166	0.0145
F_B	118.6	95.7	80.0

Case 8 $C_{U_3O_8}$ = $8/lb C_N = $100/g

	R = 0.016	0.018	0.020
y = 0.0			
$V_B(R,y)$	101.07	118.44	135.75
	0.1247	0.2819	0.3956
R_B	0.0148	0.0149	0.0148
y_B	0	0	0
F_B	93.97	76.91	64.92
y = 0.005			
$V_B(R,y)$	115.15	132.76	150.21
	0.1398	0.2971	0.4066
R_B	0.0147	0.0147	0.0147
y_B	0.00430	0.0035	0.00296
F_B	98.41	79.68	66.92
y = 0.010			
$V_B(R,y)$	126.35	144.47	162.32
	0.1229	0.2923	0.4015
R_B	0.0148	0.0147	0.0147
y_B	0.00877	0.00707	0.00598
F_B	104.4	83.42	69.72
y = 0.020			
$V_B(R,y)$	145.08	164.21	182.72
	0.0452	0.1799	0.2885
R_B	0.0155	0.0160	0.0162
y_B	0.0190	0.0164	0.0142
F_B	117.6	95.22	79.63

Fuel Value for Modified Case with Blending with Natural Uranium ($/kg U), HWR (Continued)

Case 9 $C_{U_3O_8}$ = \$6/lb C_N = \$0/g

	R = 0.016	0.018	0.020
y = 0.0			
$V_B(R,y)$	85.866	101.02	116.15
	0.04074	0.2189	0.3368
R_B	0.01563	0.0156	0.0156
y_B	0	0	0
F_B	97.31	79.41	67.26
y = 0.005			
$V_B(R,y)$	79.68	94.66	109.61
	0.0958	0.2691	0.3826
R_B	0.0151	0.0150	0.0150
y_B	0.00452	0.00365	0.00308
F_B	100.61	81.07	68.10
y = 0.010			
$V_B(R,y)$		89.22	103.84
		0.19341	0.3495
R_B		0.01586	0.01544
y_B		0.00806	0.00650
F_B		88.13	72.31
y = 0.020			
$V_B(R,y)$		81.02	95.23
		0.1159	0.2365
R_B		0.0167	0.0168
y_B		0.0176	0.01526
F_B		97.22	81.03

Case 11 $C_{U_3O_8}$ = \$6/lb C_N = \$60/g

	R = 0.016	0.018	0.020
y = 0.0			
$V_B(R,y)$	86.01	101.19	116.32
	0.0247	0.2029	0.3248
R_B	0.0157	0.0157	0.0157
y_B	0	0	0
F_B	97.87	79.97	67.68

Fuel Value for Modified Case with Blending with Natural Uranium ($/kg U), HWR
(Continued)

	R = 0.016	0.018	0.020
y = 0.005			
$V_B(R,y)$	92.21	107.45	122.59
	0.1038	0.2691	0.3826
R_B	0.0150	0.0150	0.0150
y_B	0.00448	0.00365	0.00308
F_B	100.2	81.07	68.10
y = 0.010			
$V_B(R,y)$		112.62	127.89
		0.2403	0.3695
R_B		0.0153	0.0151
y_B		0.00759	0.00630
F_B		86.05	71.35
y = 0.020			
$V_B(R,y)$		121.99	137.68
		0.1399	0.2485
R_B		0.0164	0.0167
y_B		0.0172	0.0150
F_B		96.55	80.75

APPENDIX G

G.1. Base Case Fuel Values ($/kg U), V(R,y)

Once-Through Operation of PWR. Spent Fuel Credited at its Value in HWR

$C_{U_3O_8}$ = \$8/lb C_N = \$0/g

	R = 0.02	0.025	0.03	0.04	0.05	0.06
y = 0	117.63	187.00	244.02	336.50	409.28	466.58
0.01	79.46	149.74	209.13	304.68	379.99	439.44
0.025	43.38	112.29	170.77	266.12	342.09	403.29

$C_{U_3O_8}$ = \$8/lb C_N = \$60/g

	R = 0.02	0.025	0.03	0.04	0.05	0.06
y = 0	116.80	186.46	243.91	337.09	410.20	467.17
0.01	115.65	191.34	253.52	349.93	422.69	478.51
0.025	111.90	195.39	262.82	364.94	440.07	499.55

G.2. Fuel Values for Modified Case with Pre-Enrichment by Gaseous Diffusion ($/kg U), $V_D(R,y)$

Once-Through Operation of PWR. Spent Fuel Credited at its Value in HWR

$C_{U_3O_8}$ = \$8/lb C_N = \$0/g

R =	0.005	0.010	0.015	0.020	0.025	0.030
y = 0						
$V_D(R,y)$	3.091	39.509	84.052	131.63	180.66	230.408
R_D	0.0304	0.0304	0.0304	0.0304	0.0304	0.0304
y_D	0	0	0	0	0	0
F_D	409.53	135.83	81.733	58.619	45.794	37.639
y_T	0	0	0	0	0	0
F_T	371.30	98.413	44.486	21.441	8.655	0.524
y = 0.01						
$V_D(R,y)$			61.657	105.22	151.15	198.37
R_D			0.0350	0.0344	0.0338	0.0334
y_D			0.0194	0.0153	0.0127	0.0109
F_D			116.76	78.298	58.478	46.558
y_T			0.00386	0.00324	0.00282	0.00251
F_T			70.406	34.441	15.936	4.940

Fuel Values for Modified Case with Pre-Enrichment by Gaseous Diffusion ($/kg U), $V_D(R,y)$
Once-Through Operation of PWR. Spent Fuel Credited at its Value in HWR.
(Continued)

$C_{U_3O_8}$ = \$8/lb C_N = \$60/g

	R = 0.005	0.010	0.015	0.020	0.025	0.030
y = 0						
$V_D(R,y)$	3.085	39.491	84.023	131.59	180.60	230.35
R_D	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306
y_D	0	0	0	0	0	0
F_D	409.22	135.72	81.671	58.574	45.759	37.611
y_T	0	0	0	0	0	0
F_T	371.28	98.59	44.709	21.681	8.905	0.780
y = 0.01						
$V_D(R,y)$			92.312	140.89	190.49	240.56
R_D			0.0332	0.0330	0.0326	0.0324
y_D			0.0186	0.0148	0.0123	0.0106
F_D			115.90	77.892	58.247	46.432
y_T			0.00383	0.00323	0.00280	0.00250
F_T			67.325	32.352	14.256	3.594

G.3. Fuel Value for Modified Case with Blending with Natural Uranium ($/kg U), $V_B(R,y)$

Once Through Operation of PWR. Spent Fuel Credited at its Value in HWR

$C_{U_3O_8}$ = \$8/lb C_N = \$0/g

R =	0.04	0.05	0.06	0.08	1.000	15.000
y = 0						
$V_B(R,y)$	340.03	432.49	523.21	699.62	4999.02	9413.36
	0.1818	0.3670	0.4809	0.6166	0.9465	0.9735
R_B	0.0339	0.0339	0.0339	0.0339	0.347	0.0328
y_B	0	0	0	0	0	0
F_B	26.825	20.760	16.995	12.560	1.712	0.899
y = 0.01						
$V_B(R,y)$	306.22	397.10	486.50	660.64	4916.20	9285.53
	0.1197	0.3252	0.4544	0.5989	0.9459	0.9732
R_B	0.0359	0.0356	0.0352	0.0350	0.0347	0.0328
y_B	0.00880	0.00678	0.00546	0.00401	0.00054	0.00027
F_B	32.188	23.993	19.162	13.774	1.748	0.914
y = 0.025						
$V_B(R,y)$	266.17	352.41	438.40	607.46	4792.60	9094.16
	0.0245	0.2720	0.4159	0.5763	0.9451	0.9728
R_B	0.0391	0.0378	0.0370	0.0363	0.0347	0.0327
y_B	0.0244	0.0182	0.0146	0.0106	0.00137	0.00068
F_B	40.069	29.271	22.793	15.784	1.804	0.936

Fuel Value for Modified Case with Blending with Natural Uranium ($/kg U), $V_B(R,y)$
Once-Through Operation of PWR. Spent Fuel Credited at its Value in HWR

(Continued)

$C_{U_3O_8}$ = \$8/lb C_N = \$60/g

	R = 0.04	0.05	0.06	0.08	1.000	15.000
y = 0.0						
$V_B(R,y)$	340.29	432.84	523.64	700.19	5004.05	9418.11
	0.1738	0.3590	0.4769	0.6126	0.9465	0.9735
R_B	0.0341	0.0342	0.0341	0.0342	0.0346	0.0328
y_B	0	0	0	0	0	0
F_B	26.855	20.790	17.010	12.575	1.712	0.8995
y = 0.01						
$V_B(R,y)$	352.24	444.09	534.12	709.07	4970.45	9339.86
	0.1317	0.3331	0.4544	0.5989	0.9459	0.9732
R_B	0.0355	0.0352	0.0352	0.0350	0.0346	0.0328
y_B	0.00868	0.00667	0.00546	0.00401	0.00541	0.00268
F_B	32.128	23.953	19.162	13.774	1.748	0.914
y = 0.025						
$V_B(R,y)$	365.94	457.76	547.27	720.55	4919.74	9222.38
	0.0885	0.3040	0.4319	0.5843	0.9451	0.9728
R_B	0.0369	0.0364	0.0362	0.0358	0.0347	0.0329
y_B	0.0228	0.0174	0.0142	0.0104	0.00137	0.00068
F_B	41.167	29.345	22.767	15.751	1.804	0.936

APPENDIX H

NOMENCLATURE

B	average burnup, megawatt days/metric ton
C_A	unit cost of reprocessing, including conversion of UNH to UO_3, \$/kg fuel fed to reprocessing
$C_{AEC}(R)$	price of UF_6 with zero U-236 content and with abundance ratio R, based on the AEC scale, \$/kg U
C_C	unit cost of converting UO_3 to UF_6, \$/kg U fed to conversion
C_{CT}	cost incurred between purchase of UO_3 and end of conversion to UF_6, excluding inventory charges, \$/kg U fed to conversion
$C_E(R)$	fuel cycle cost when feed having abundance ratio R and zero U-236 content is purchased as UF_6 on the AEC scale, mills/kwhr
C_E^*	minimum fuel cycle cost realizeable when feed having zero U-236 content is purchased as UF_6 on the AEC price scale, mills/kwhr
C_F	unit cost of fabrication, including conversion of UO_3 or UF_6 to UC, \$/kg U leaving fabrication
C_K	unit price of fissile plutonium, \$/g
C_N	unit price of Np-237, \$/g
$C_N^o(R,y)$	the indifference value for Np, i.e. that value at which the penalty for U-236 equals zero

C_{NAT} the cost of natural uranium as UF_6 on the AEC price scale, \$/kg U

C_{POST} the cost of reprocessing plus shipping minus credit for plutonium and neptunium, or cost of storage in lieu of reprocessing, whichever is smaller, \$/kg U

$C_S(R_S, y_S)$ the credit for spent uranium from PWR of composition R_S, y_S, \$/kg U

C_{SH} unit shipping cost for irradiated fuel, \$/kg fuel shipped

C_{STOR} the cost of storage in lieu of reprocessing, \$/kg U

$C_{U_3O_8}$ price of natural uranium as U_3O_8, \$/lb U_3O_8

C_Δ cost of separative work, \$/kg U

$C_\$(R)$ fuel cycle cost when feed having abundance ratio R and zero U-236 content is purchased as UF_6 on the AEC scale, \$/kg U

F time-averaged flow rate of uranium fed to fabrication, kg U/day

F_B time-averaged flow rate of uranium to be blended with natural uranium, kg U/day

F_D time-averaged flow rate of uranium to be pre-enriched by gaseous diffusion, kg U/day

F_{NAT} time-averaged flow rate of natural uranium to be used in blending, kg U/day

F_R time-averaged flow rate fed to the reactor, kg U/day

F_S time-averaged flow rate of uranium leaving the reprocessing plant, kg U/day

F_T time-averaged flow rate of uranium in the tails stream from the diffusion plant used for pre-enrichment, kg U/day

i annual charge rate on working capital, yr^{-1}

I initial uranium loading of reactor, kg

K time-averaged flow rate of fissile plutonium leaving reprocessing plant, kg/day

L average load factor for power plant

L_C fractional loss of uranium during chemical conversion of UO_3 or U_3O_8 to UF_6, based on product from conversion

L_F fractional loss of uranium during fabrication, based on material leaving fabrication

L_{RP} fractional loss of Pu and Np during reprocessing, based on material fed to the reprocessing plant

L_{RU} fractional loss of uranium during reprocessing, based on uranium fed to the reprocessing plant

N time-averaged flow rate of Np-237 leaving reprocessing plant, kg/day

P net electrical power output of plant, MW(e)

Q_K the number of grams of fissile plutonium discharged from the reactor per initial kilogram of uranium

Q_N the number of grams of Np-237 discharged from the reactor per initial kilogram of uranium

R weight ratio of U-235 to U-238 in uranium for which the value is to be determined

R^* weight ratio of U-235 to U-238 which gives minimum fuel cycle cost when makeup feed having zero U-236 content is purchased as UF_6 on the AEC price scale.

R_B weight ratio of U-235 to U-238 in product stream from blending

R_D weight ratio of U-235 to U-238 in head stream from diffusion plant used for pre-enriching

R_{NAT} U-235 to U-238 weight ratio for natural uranium

R_R weight ratio of U-235 to U-238 in stream fed to the reactor

R_S weight ratio of U-235 to U-238 in uranium stream leaving the reprocessing plant

R_T weight ratio of U-235 to U-238 in tails stream from diffusion plant used for pre-enriching

t_C time interval between purchase of UO_3 or U_3O_8 and completion of conversion to UF_6, years

t_E time interval between the delivery of uranium to the AEC for toll enrichment and the receipt of product uranium, years

t_F average pre-reactor fuel holdup time, years

t_R reactor residence time, years

t_{RP} average post-reactor holdup time for Pu and Np, years

t_{RU} average post-reactor holdup time for uranium, years

$V(R,y)$ unit fuel value of UO_3 having composition R,y when used as feed in the base case mode of operation, \$/kg U

$V_B(R,y)$ maximum unit fuel value of UO_3 having composition R,y which is attainable when it is blended with natural uranium, \$/kg U

$V_D(R,y)$ maximum unit fuel value of UO_3 having composition R,y which is attainable when it is pre-enriched by gaseous diffusion, \$/kg U

$V_m(R,y)$ the largest of $V(R,y)$, $V_B(R,y)$, and $V_D(R,y)$ for a given isotopic composition, \$/kg U

y weight fraction of U-236 in uranium for which the value is to be determined

y_B weight fraction of U-236 in product stream from blending

y_D weight fraction of U-236 in heads stream from diffusion plant used for pre-enriching

y_R weight fraction of U-236 in stream fed to the reactor

y_S weight fraction of U-236 in uranium stream leaving the reprocessing plant

y_T weight fraction of U-236 in tails stream from diffusion plant used for pre-enriching

ε weight fraction of natural uranium used in blending

Δ_D separative work requirement for the pre-enrichment of feed uranium, kg U/day

δ U-236 penalty, defined by Equation VII.1

η thermal efficiency

APPENDIX I

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