MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF NUCLEAR ENGINEERING
Cambridge 39, Massachusetts

NYO-10,208 MITNE-26

AEC Research and Development Report
UC-34 Physics
(TID-4500, 18th Edition)

HEAVY WATER LATTICE PROJECT ANNUAL REPORT

September 30, 1962

Contract AT(30-1)2344
U.S. Atomic Energy Commission

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UC-34 Physics

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ABSTRACT

An experimental and theoretical program on the physics of heavy water-moderated, partially enriched uranium lattices is being conducted at the Massachusetts Institute of Technology. Experimental methods have been adapted or developed for research on buckling, fast fission, resonance capture, and thermal capture. They have been applied successfully to three lattices of one-inch diameter, natural uranium rods in heavy water, which served as calibration lattices. Some new and useful information has also been obtained for these lattices. Research on lattices of enriched uranium rods is in progress. Programs are under way in pulsed neutron source research and reactor control research.

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1. INTRODUCTION

This report is the second annual progress report of the Heavy Water Lattice Project of the Massachusetts Institute of Technology. It covers the period from October 1, 1961 to September 30, 1962. The origin, early history, and main facilities of the project have been described in the first progress report (NYO-9658, September 30, 1961). The present report summarizes the research completed up to September 30, 1962. The details of this research are given in the following reports and papers published during the past year:

1. J. T. Madell, T. J. Thompson, A. E. Profio, and I. Kaplan, Spatial Distribution of the Neutron Flux on the Surface of a Graphite-Lined Cavity, NYO-9657, April, 1962.

2. A. Weitzberg, I. Kaplan, and T. J. Thompson, Measurements of Neutron Capture in U^{238} in Lattices of Uranium Rods in Heavy Water, NYO-9659, January 8, 1962.

3. P. F. Palmedo, I. Kaplan, and T. J. Thompson, Measurements of the Material Bucklings of Lattices of Natural Uranium Rods in D_2O, NYO-9660, January 20, 1962.

4. J. R. Wolberg, T. J. Thompson, and I. Kaplan, A Study of the Fast Fission Effect in Lattices of Uranium Rods in Heavy Water, NYO-9661, February 21, 1962.

5. J. Peak, I. Kaplan, and T. J. Thompson, Theory and Use of Small Subcritical Assemblies for the Measurement of Reactor Parameters, NYO-10204, April 9, 1962.

6. P. S. Brown, T. J. Thompson, I. Kaplan, and A. E. Profio, Measurements of the Spatial and Energy Distribution of Thermal Neutrons in Uranium, Heavy Water Lattices, NYO-10205, August 20, 1962.

7. J. T. Madell, T. J. Thompson, I. Kaplan, and A. E. Profio, Calculation of the Flux Distribution in a Cavity Assembly, Trans. Amer. Nuclear Soc. 5, (June, 1962) p. 85.

8. A. Weitzberg, J. R. Wolberg, T. J. Thompson, A. E. Profio, and I. Kaplan, Measurements of U^{238} Capture and Fast Fission in Natural Uranium, Heavy Water Lattices, Trans. Amer. Nuclear Soc. 5 (June, 1962) p. 86.

9. P. S. Brown, P. F. Palmedo, T. J. Thompson, A. E. Profio, and I. Kaplan, Measurements of Microscopic and Macroscopic Flux Distributions in Natural Uranium, Heavy Water Lattices, Trans. Amer. Nuclear Soc. 5 (June, 1962) p. 87.

10. P. S. Brown, I. Kaplan, A. E. Profio, and T. J. Thompson, Measurements of the Spatial and Spectral Distribution of Thermal Neutrons in Natural Uranium Heavy Water Lattices, Brookhaven Conference on Neutron Thermalization, April 30 - May 2, 1962.

11. K. F. Hansen, Multigroup Diffusion Methods, NYO-10206, April, 1962.

The work described in detail in the reports and papers listed above will be summarized briefly in the present reports; the major emphasis will be placed on work which has not yet been published.

The project staff, as of September, 1962, is as follows:

I. Kaplan, Professor of Nuclear Engineering
T. J. Thompson, Professor of Nuclear Engineering
A. E. Profio, Research Associate
D. D. Lanning, Assistant Professor of Nuclear Engineering

P. S. Brown, DSR Staff
B. K. Malaviya, Research Assistant
S. G. Oston, AEC Fellow
R. Simms, AEC Fellow
J. Gosnell, Research Associate (not charged to contract)
W. H. D'Ardenne, AEC Fellow
J. Harrington, III, AEC Fellow
J. H. Barch, Senior Technician
A. T. Supple, Jr., Technician
Miss B. Kelley, Technical Assistant
D. A. Gwinn, Technician (part time)
J. Canosa, DSR Staff (summer only)
C. Ribbeck, Research Assistant (summer only)
Miss D. M. Dutton, Secretary (part time)

2. THE RESEARCH PROGRAM

The general objective of the lattice project is to carry out experimental and theoretical investigations of the physics of subcritical lattices of partially enriched uranium rods in heavy water. An initial study has been made with natural uranium rods, 1.0 inch in diameter, in triangular lattices with spacings of 4.5 inches, 5 inches, and 5.75 inches, respectively. These lattices have been used to test the accuracy of the methods used and to obtain new information. Most of the results to be reported were obtained with these lattices. In addition, a start has been made on lattices of 0.25-inch diameter rods containing U^{235} at a concentration of 1.03 per cent; measurements have been made so far in a triangular lattice with a spacing of 1.25 inches. During the coming year, the lattice spacing will be varied and the experiments extended. The plans for the general program include the study of rods containing U^{235} at concentrations between one and two per cent and having diameters from 0.25 inch to 0.75 inch. The ratio of volume of heavy water to that of uranium is to be varied, with values in the range 10 to 90. The following materials are either available or in the process of fabrication, in addition to the normal uranium rods: 0.25-inch diameter rods, of 1.03% U^{235} uranium metal; 0.25-inch diameter rods of 1.14% U^{235} uranium metal; 0.4-inch diameter rods, of 1.1% U^{235} uranium oxide; 0.4-inch diameter rods, of 2% U^{235} uranium oxide; 0.75-inch diameter rods, of 0.95% U^{235} uranium metal, and 0.387-inch diameter rods, of 0.95% U^{235} uranium metal.

The measurements made include those of the following quantities: macroscopic radial and axial thermal neutron flux traverses, giving the critical buckling; activation ratios related to the fast fission effect; quantities related

to the capture of resonance neutrons, including the ratio of U^{238} captures to U^{235} fissions in the fuel rod, the average U^{238} cadmium ratio in the fuel rod, and quantities related to the effective resonance integral of the rods; intracell thermal neutron flux distribution and effective neutron temperatures. Studies have also been started on the determination of absolute fission and capture rates. In addition to the main subcritical (exponential) assembly, fed by thermal neutrons from the thermal column of the MITR, a "miniature lattice" has been used to study some of the quantities mentioned above, as well as the theory of exponential assemblies. The purpose of these experiments is to explore the possible use of lattices containing few rods, or very short rods; results obtained in miniature lattices are to be compared with those obtained in the larger exponential assemblies. Plans are also being made for studies of single rods in various moderators.

Studies are also under way with pulsed neutron sources, with the object of supplementing and extending steady-state lattice experiments; experiments on reactivity and control rod effects have been started. Plans are also being made to extend the research program to include experimental studies of the neutron energy spectra in the lattices under investigation.

The experimental work has two objectives: (1) to improve existing methods and develop new ones where possible, and (2) to apply the methods to different lattices. The emphasis is on precise, theoretically interpretable experiments. A program of theoretical research is carried on, parallel to the experimental work. This program also has two objectives: (1) to relate the experimental results to existing theory, and (2) to extend the theory where possible.

3. FACILITIES

The major experimental facility, the exponential assembly and its ancillary equipment, has been described in NYO-9658. The small subcritical assembly, or "miniature lattice" facility, has been described in NYO-10204. The pulsed neutron source has been described in NYO-9658; some additional information concerning this facility is given in section 10 below.

4. THE MATERIAL BUCKLING

4.1 LATTICES WITH NATURAL URANIUM (P. F. Palmedo*)

The details of the methods used and the results obtained for the natural uranium, heavy water lattices are given in NYO-9660 by Palmedo et al. (cf. Introduction, ref. 3). The results may be summarized as follows:

1. Methods were investigated for adjusting the distribution and magnitude of the flux entering the experimental tank. The shape and magnitude of that flux could be varied by changing the configuration of the pedestal supporting the tank. A configuration was devised that provided an incoming flux appropriate for both macroscopic and microscopic (intracell) measurements in lattices.

2. Experimental and analytical methods were developed which were found to be easy to use and productive of accurate results. Computer codes, developed for handling the experimental data, were easily adapted to the various experimental methods used and efficient and accurate in deriving the buckling from measured flux traverses. The variety and flexibility of the codes made them well suited to the investigation of details of the flux distribution and of the fitting process.

3. The experiments showed that the macroscopic flux distribution, except for intracell effects, had a simple form throughout most of the system. The flux could be described by a single J_o function radially, and by a single hyperbolic sine function axially. Reflector effects were negligible, and macroscopic energy-space separability, as based on epicadmium and subcadmium traverses, held throughout most of the volume of

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*Now at Centre d'Études de Saclay, France

the lattices studied. A very small deviation was observed, only a few per cent in magnitude, from separability of the macroscopic and microscopic distributions in the radial direction.

4. The values of the buckling were determined for three lattices of natural uranium rods 1.0 inch in diameter, clad in 0.028-inch thick aluminum. The results are listed in Table 4.1.

TABLE 4.1

Critical Buckling of Lattices of 1.0-Inch Diameter Rods in Heavy Water

Lattice Spacing (Inches)	Ratio of Moderator Volume to Fuel Volume	Radial Buckling (m^{-2})	Axial Buckling (m^{-2})	Critical Buckling (m^{-2})
4.5	21.0	14.11 ± 0.06	5.63 ± 0.08	8.48 ± 0.10
5.0	26.4	14.12 ± 0.06	5.47 ± 0.08	8.65 ± 0.10
5.75	35.6	14.20 ± 0.05	6.05 ± 0.05	8.15 ± 0.08

They are also plotted in Fig. 4.1, which represents a compilation of buckling results obtained at a number of laboratories, over several years, in both critical and exponential experiments. The curve drawn through the points is based mainly on the Savannah River and North American results. The M.I.T. measurements are seen to be in good agreement with the other results when the uncertainties of the measurements are taken into account.

4.2 LATTICES WITH ENRICHED URANIUM
(J. Harrington, III)

Measurements of axial and radial traverses have recently been completed in the first lattice of 0.25-inch diameter rods (clad in 0.028-inch thick aluminum), containing 1.03 per cent U^{235}, in a triangular lattice with a spacing of 1.25 inches. The analysis of the data has not yet been finished.

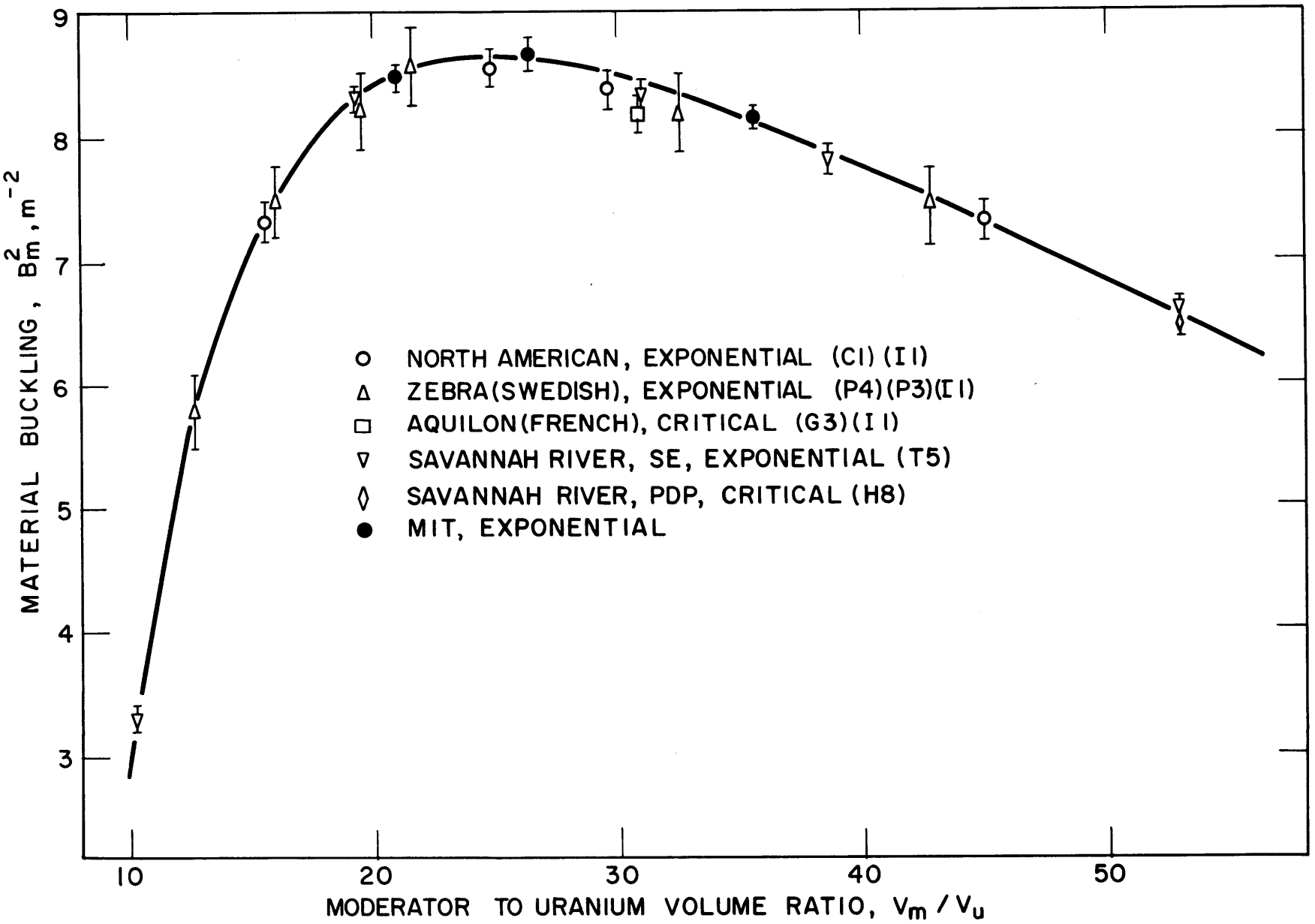

FIG. 4.1 BUCKLINGS OF 1.0 INCH DIAMETER, NATURAL URANIUM RODS IN D_2O – ALL MEASUREMENTS CORRECTED TO 99.75 MOLE % D_2O

5. FAST FISSION MEASUREMENTS AND RELATED STUDIES

5.1 LATTICES WITH NATURAL URANIUM
(J. R. Wolberg*)

The details of the methods used and the results obtained for the natural uranium, heavy water lattices are given in NYO-9661 by Wolberg et al. (cf. Introduction, ref. 4). The results may be summarized as follows.

1. A new method has been developed for the measurement of δ_{28}, the ratio of the fission rate in U^{238} to the fission rate in U^{235}, and the effects of variations in the experimental conditions on the measurement of δ_{28} have been investigated. The new method involves the determination of the ratio of the 1.60 Mev-La^{140} activity in uranium foils of different U^{235} concentration.

2. The development of the new method has involved careful studies of the fission product gamma-ray spectrum as a function of time after irradiation, for gamma rays up to 2.7 Mev.

3. The new method for measuring δ_{28} involves an irradiation within a fuel rod, without requiring a supplementary fission chamber experiment. The uncertainty associated with the method is smaller than that of earlier methods, the major uncertainty being in the ratio (β^{25}/β^{28}) for La^{140}, where the β's represent fission product yields. The results obtained can be corrected as better data on fission product yields become available.

4. Measurements of δ_{28} have been made for a single natural uranium rod 1.0 inch in diameter in heavy water, and

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*Now at Technion Institute, Israel

for three lattices of such rods in heavy water. The spacings of the rods, in triangular lattices, were 4.5, 5, and 5.75 inches, respectively. The results of the measurements are listed in Table 5.1 below.

The value of δ_{28} measured in the tightest lattice studied was only slightly over six per cent greater than the value obtained for a single rod, indicating that the fast interaction effects in the lattices studied were small.

5. Measurements have been made of δ_{25}, the ratio of epicadmium fission rate to subcadmium fission rate in U^{235}. The results for a single rod and for the three lattices of natural uranium rods in heavy water are listed in Table 5.1 below, together with values of δ_{25} calculated from measured values of the gold cadmium ratio.

TABLE 5.1

Values of δ_{28} and δ_{25} in Lattices of 1.0-Inch Diameter Uranium Rods in Heavy Water

Lattice Spacing (Inches)	Ratio of Moderator Volume to Fuel Volume	δ_{28}	δ_{25} (Directly Measured)	δ_{25} (Calculated from Gold Cadmium Ratios)
4.5	21.0	0.0597 ± 0.0020	0.0479 ± 0.0019	0.048
5.0	26.4	0.0596 ± 0.0017	0.0340 ± 0.0030	0.041
5.75	35.6	0.0583 ± 0.0012	0.0268 ± 0.0010	0.026
Single Rod		0.0559 ± 0.0015	0.0086 ± 0.0004	
Moderator			0.0035 ± 0.0002	

The measurements of δ_{25} for the single rod and the moderator were used to find where the neutrons causing epicadmium fission were born. In the 5.75-inch lattice, for example, at a

position one foot above the bottom of the tank, approximately 13% $\left(\frac{0.0035}{0.0268}\right)$ of the epicadmium fissions in U^{235} were caused by neutrons entering the tank from the thermal column; 19% $\left(\frac{0.0086-0.0035}{0.0268}\right)$ were caused by neutrons born in the rod, and the remaining 68% were caused by neutrons born in other rods. The fraction of epicadmium fissions caused by neutrons born in other rods in a lattice increases with decreasing values of the ratio of moderator volume to fuel volume because of the hardening of the neutron energy spectrum.

6. The value of δ_{28} has also been measured for a single uranium rod 0.25 inch in diameter with a U^{235} concentration of 1.143 per cent, immersed in heavy water: the result obtained was 0.0126 ± 0.0004.

5.2 LATTICES WITH ENRICHED URANIUM

(W. H. D'Ardenne)

The fast fission effect and related quantities have also been measured in the first lattice of enriched uranium rods (0.25-inch diameter rods, 1.03 per cent U^{235}, with a spacing of 1.25 inches) in heavy water. The experimental data are being analyzed.

6. STUDIES OF RESONANCE CAPTURE IN U^{238}

6.1 LATTICES WITH NATURAL URANIUM (A. Weitzberg*)

The details of the methods used and the results obtained for the natural uranium, heavy water lattices are given in NYO-9659 by Weitzberg et al. (cf. Introduction, ref. 2). The study included the following experiments:

1. Measurement of the average U^{238} cadmium ratio of the fuel rods, R^{28}.

2. Measurement of the average ratio of the U^{238} capture rate to the U^{235} fission rate in the fuel rods. This ratio, C^*, is the experimentally determined conversion ratio.

3. Measurement of the distribution of resonance neutrons in the moderator; both macroscopic and intracell distributions were measured.

4. Measurement of quantities related to the effective resonance integral of the rods.

The results of the measurements of the cadmium ratio and the conversion ratio are listed in Table 6.1, together with the value of ρ_{28}, the ratio of epicadmium to subcadmium capture in U^{238}, given by $\rho_{28} = (R_{28}-1)^{-1}$. The initial conversion ratio C for a lattice is usually defined as the ratio of the capture rate in U^{238} to the absorption rate in U^{235}. It can be calculated from C^*, the measured conversion ratio, or independently from R^{28}, the measured U^{238} cadmium ratio. Thus, C is related to C^* by the equation

$$C = C^*(1+\bar{a})^{-1},$$

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*Now at Atomics International, Canoga Park, California

TABLE 6.1

Values of the Cadmium Ratio and Conversion Ratio for Lattices of 1.0-Inch Diameter Natural Uranium Rods in Heavy Water

Lattice Spacing (Inches)	Ratio of Moderator Volume to Fuel Volume	U^{238} Cadmium Ratio (R_{28})	Ratio of Epithermal U^{238} Capture to Thermal U^{238} Capture $\left(\rho^{28}=\frac{1}{R_{28}-1}\right)$	Conversion Ratio $C^*=\frac{U^{238}\text{ Capture}}{U^{235}\text{ Fission}}$
4.5	21.0	2.972 ± 0.031	0.507 ± 0.008	1.017 ± 0.023
5.0	26.4	3.495 ± 0.010	0.401 ± 0.002	0.948 ± 0.020
5.75	35.6	4.223 ± 0.043	0.310 ± 0.004	0.859 ± 0.016

where $\bar{\alpha}$ is the average value of the ratio of capture to fission in the fuel. The conversion ratio C is also related to R_{28} by the equation:

$$C = \frac{1+\rho_{28}}{1+\rho_{25}}\left(\frac{\Sigma_a^{28}}{\Sigma_a^{25}}\right),$$

where ρ_{25} is the ratio of epicadmium to subcadmium absorption in U^{235}, and $\left(\Sigma_a^{28}/\Sigma_a^{25}\right)$ is the ratio of the absorption rate in U^{238} to that in U^{235}, below the cadmium cutoff. The values of $\bar{\alpha}$ and $\left(\Sigma_a^{28}/\Sigma_a^{25}\right)$ were obtained by means of the THERMOS code (see section 7 below), and were found to be

$$1+\bar{\alpha} = 1.174 \pm 0.010,$$

$$\left(\Sigma_a^{28}/\Sigma_a^{25}\right) = 0.574 \pm 0.006.$$

The values of δ_{25} were independently obtained from fission measurements on U^{235} foils and from cadmium ratios for thin

gold foils; the values are listed in Table 5.1.

The values for the conversion ratio, C, are listed in Table 6.2.

TABLE 6.2

Values of the Initial Conversion Ratios for Lattices of 1.0-Inch Diameter Uranium Rods in Heavy Water

Lattice Spacing (Inches)	Ratio of Moderator Volume to Uranium Volume	Initial Conversion Ratio, C	
		From Measured Value of C* $\left(\frac{U^{238}\ \text{Absorption}}{U^{235}\ \text{Fission}}\right)$	From U^{238} Cadmium Ratio, R_{28}
4.5	21.0	0.866 ± 0.021	0.826 ± 0.010
5.0	26.4	0.808 ± 0.018	0.775 ± 0.009
5.75	35.6	0.732 ± 0.018	0.733 ± 0.008

The results obtained for the conversion ratio by means of the two independent methods are in reasonably good agreement. A small discrepancy, slightly greater than the experimental uncertainty, appears for the tighter lattices. It was considered too small to warrant further study in these lattices, but investigations of the discrepancy are being continued in the lattices of partially enriched uranium rods in heavy water.

Measurements of the resonance neutron distributions were made with cadmium-covered, dilute, natural uranium foils; together with the results of similar measurements made with bare and cadmium-covered gold foils, they served to demonstrate the separability of the energy and spatial dependence of the flux over a considerable portion of the assembly. This result insured that the measurements of the intracell parameters were made in a neutron energy spectrum representative of the lattice.

A new technique was developed to measure the effective resonance integral of the fuel rods, because standard single-rod methods were found to be inadequate for measurements in a lattice. Instead of comparing the epicadmium activation of the fuel rod to that of a dilute U^{238} standard, the rod cadmium ratio was compared with the cadmium ratios of Na^{23} (a 1/v absorber), and Mn^{55} and Co^{59} whose principal resonances are at 332 and 137 ev, respectively. The standardization was consequently insensitive to perturbations in the 1/E flux distribution which occur at energies corresponding to the large U^{238} resonances, i.e., at energies much smaller than 138 ev.

6.2 LATTICES WITH ENRICHED URANIUM (W. H. D'Ardenne)

Measurements of the absorption of epicadmium neutrons in U^{238} and related quantities have been started in the first lattice of enriched uranium rods (0.25-inch diameter, 1.03 per cent U^{235}, with a spacing of 1.25 inches) in heavy water. It is, as yet, too early to report any results.

7. INTRACELL THERMAL NEUTRON DISTRIBUTIONS AND THE CAPTURE OF THERMAL NEUTRONS IN LATTICES OF NATURAL URANIUM RODS IN HEAVY WATER AND IN A LATTICE OF PARTIALLY ENRICHED URANIUM RODS IN HEAVY WATER

(P. S. Brown)

Measurements of intracell thermal neutron distributions and associated quantities have been completed for three lattices of natural uranium rods in heavy water and for one lattice of partially enriched uranium rods in heavy water. The details of the methods used and the results obtained are given in NYO-10205, by Brown et al. (cf. Introduction, ref. 6). The results are summarized briefly.

7.1 NEUTRON DENSITY DISTRIBUTIONS

Thermal neutron density distributions were measured with gold foils in three natural uranium, heavy water lattices of 1.0-inch diameter, aluminum clad rods on triangular spacings of 4.5, 5.0, and 5.75 inches, respectively. The measured distributions were compared with distributions computed using the THERMOS code (1) developed by H. C. Honeck for the numerical solution of the integral transport equation for the thermal neutron density in a lattice cell. The agreement between the theoretical (THERMOS) and experimental distributions was excellent in the cell and in the moderator as shown, for example, in Fig. 7.1 for the lattice with triangular spacing of 5.75 inches.

Thermal neutron density distributions were measured with three kinds of gold detector foils in a lattice of 0.25-inch diameter, 1.03% U^{235}, aluminum clad rods on a triangular spacing of 1.25 inches. The three detectors were: 1/16-inch diameter pure gold foils, 1/32-inch diameter gold foils, and 1/16-inch diameter

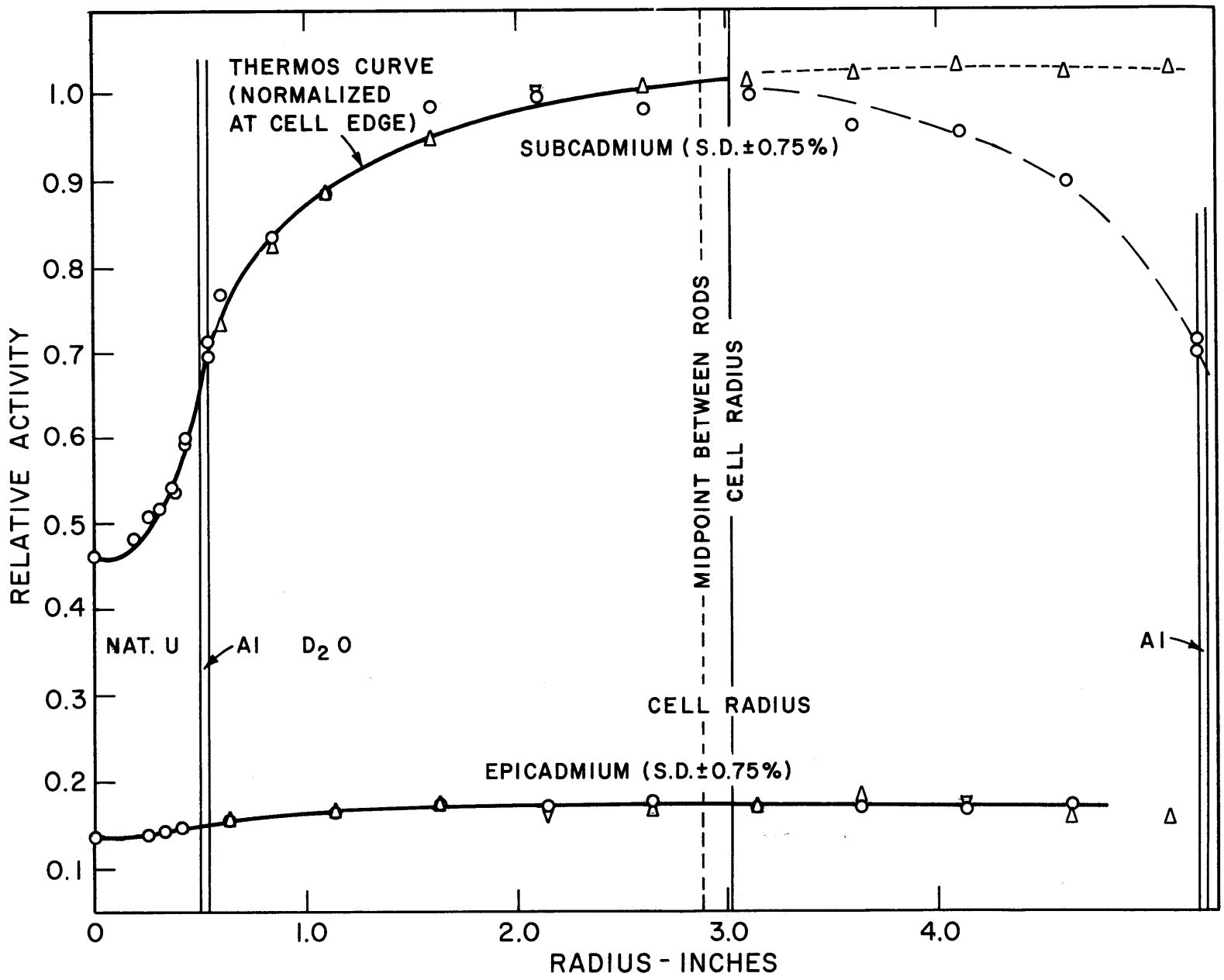

FIG. 7.1 GOLD ACTIVITY DISTRIBUTION, 1.010 - In. - Dia. Nat. U. RODS ON A 5.75 - In. TRIANGULAR SPACING.

foils of a lead-gold alloy which closely matched the absorption and scattering properties of the fuel. The distributions measured with the three detectors were the same, within the experimental uncertainties. The three detectors were chosen because they cause different amounts of flux perturbation. Since all three detectors gave the same results, the corrections used for the flux perturbations seemed to be adequate.

The neutron density distribution in the lattice of enriched rods was compared with the distribution computed with THERMOS. When the THERMOS results were normalized in the fuel, the theoretical distribution agreed well with the measured distribution in the fuel, but was considerably higher than the measured distribution in the moderator. With normalization at the cell edge, the theoretical distribution was 13.2% lower than the measured distribution at the cell center (cf. Fig. 7.2). Attempts to resolve this discrepancy will be discussed below.

7.2 SPECTRAL EFFECTS ON SPATIAL THERMAL NEUTRON DISTRIBUTIONS

Intracell activity distributions were obtained with bare and cadmium-covered foils of lutetium and europium. The lutetium isotope Lu^{176} has a strong neutron absorption resonance at 0.142 ev, which makes the activation of lutetium sensitive to the shape of the neutron energy spectrum in the energy region where the 1/E flux joins the thermal distribution. The europium isotope Eu^{151} has a thermal neutron absorption cross section which varies roughly as $1/v^2$, a strong capture resonance at 0.46 ev, and a weaker resonance at 0.32 ev which slightly distorts the shape of the resonance at 0.46 ev. The $1/v^2$-behavior of the absorption cross section of Eu^{151} implies that the activation of europium is sensitive to the shape of the low-energy region of the neutron energy spectrum. A method was developed for treating the partial absorption, by cadmium covers, of neutrons at the 0.46 ev resonance of europium, and it was found possible to correct the

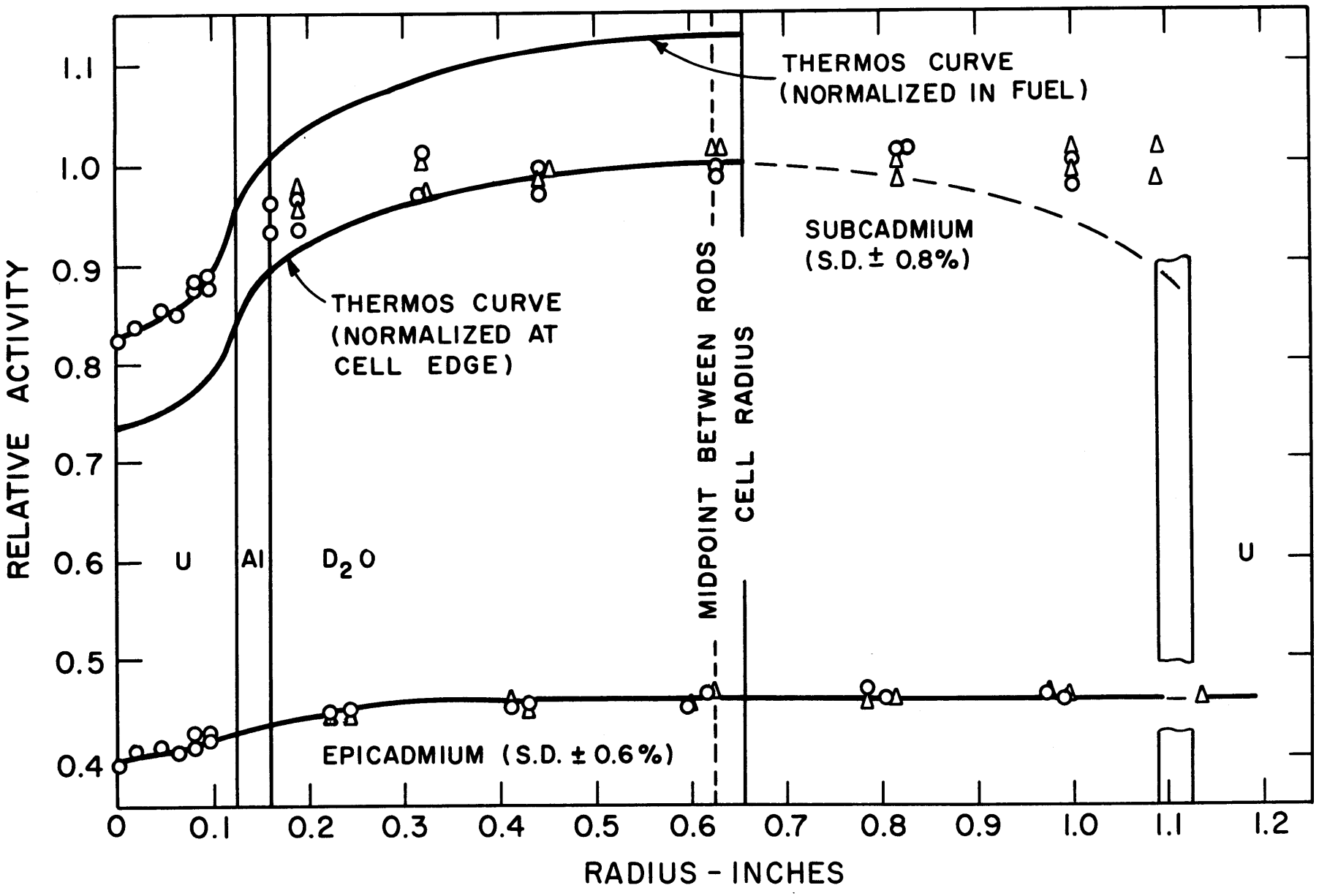

FIG. 7.2 GOLD ACTIVITY DISTRIBUTION (1/16-IN. DIA. FOILS), 0.250-IN. DIA. RODS OF 1.03% U^{235} ON A 1.25-IN. TRIANGULAR SPACING

europium activity distributions to energy cutoffs just above, and just below, the resonance.

With the three detectors, gold, lutetium, and europium, it was possible to investigate, by means of integral measurements, the gross spectral dependence of the intracell thermal neutron distributions.

In the natural uranium lattices, the measured distributions agreed well with the theoretical (THERMOS) distributions for both lutetium and europium, although the agreement was not quite as good as that in the case of gold activation. The THERMOS calculations, therefore, predict well the degree of spectral hardening from moderator to fuel. In the enriched lattice, discrepancies were observed between the distributions predicted by THERMOS and those measured similar to the discrepancy observed in the case of gold activation. However, the measured ratios of lutetium activation to gold activation, and of europium activation to gold activation, agreed well with those computed with THERMOS. The agreement of the activity ratios indicates that THERMOS gives a good estimate of the degree of spectral hardening, even though the estimated neutron density itself may be in error.

Neutron temperature changes from the cell edge to the cell center and from the cell edge to the rod surface were calculated from the measured gold, lutetium, and europium activity distributions in the natural and enriched lattices; Westcott effective cross sections were used in these calculations. The temperature changes based on the lutetium distributions agreed well with those based on the europium distributions, within the experimental uncertainties. The Westcott g-factor for europium varies much less with temperature than that for lutetium with the result that europium proved to be a less sensitive spectral indicator than lutetium. The neutron temperature changes obtained with europium had, therefore, much greater uncertainties than those obtained for lutetium. The neutron temperature calculations

based on the Westcott formulation yielded results that agreed well with those obtained from THERMOS calculations, indicating that the Westcott method may still be a useful tool, even though it approximates only roughly the actual shape of the spectral distribution and does not take into account spectral hardening in the fuel.

7.3 DISADVANTAGE FACTORS AND THERMAL UTILIZATION

The disadvantage factors measured in the natural uranium lattices agreed well with those computed with THERMOS. The method of Amouyal, Benoist and Horowitz (ABH), (2), with cross sections derived from THERMOS also gave disadvantage factors that agreed well with experiment, and with the values obtained with THERMOS. Consequently, the values obtained for the thermal utilization, f, by means of the three methods agreed well. Some of the results are listed in Tables 7.1 and 7.2.

TABLE 7.1

Thermal Neutron Density Disadvantage Factors for Natural Uranium Lattices

Lattice Spacing (Inches)	$\frac{V^*_{Mod}}{V_{Fuel}}$	Moderator-to-Fuel Disadvantage Factor, $\bar{N}(Mod)/\bar{N}(Fuel)$		Fuel Disadvantage Factor, $\bar{N}(Clad)/\bar{N}(Fuel)$	
		Experiment	THERMOS	Experiment	THERMOS
4.5	21.0	1.720±0.017	1.734	1.254±0.015	1.266
5.0	26.4	1.721±0.022	1.760	1.248±0.019	1.268
5.75	35.6	1.796±0.012	1.795	1.247±0.011	1.271

TABLE 7.2

Values of the Thermal Utilization for the Natural Uranium Lattices (99.73% D_2O)

Lattice Spacing (Inches)	$\frac{V_{Mod}}{V_{Uranium}}$	Thermal Utilization, f		
		Experiment	THERMOS	ABH
4.5	21.0	0.9855 ± 0.0006	0.9849 ± 0.0006	0.9851 ± 0.0006
5.0	26.4	0.9836 ± 0.0007	0.9828 ± 0.0007	0.9830 ± 0.0007
5.75	35.6	0.9799 ± 0.0010	0.9792 ± 0.0010	0.9793 ± 0.0010

In the enriched lattice, the measured values of the disadvantage factors and thermal utilization do not agree with those predicted by THERMOS, as can be seen from Tables 7.3, 7.4 and 7.5. The probable reason for the discrepancies will be discussed in the next section.

TABLE 7.3

Neutron Density Disadvantage Factors for the Enriched Lattice (0.25-Inch, 1.03% U^{235} Rods with a Triangular Spacing of 1.25 Inches)

	THERMOS	Experiment
$\frac{\bar{N}_{mod}}{\bar{N}_{fuel}}$	1.262	1.138 ± 0.011
$\frac{\bar{N}_{clad}}{\bar{N}_{fuel}}$	1.124	1.073 ± 0.016
$\frac{\bar{N}_{fuel\ surface}}{\bar{N}_{fuel}}$	1.099	1.059 ± 0.016

TABLE 7.4

Flux Disadvantage Factors for the Enriched Lattice

	THERMOS	ABH	Experiment
$\frac{\bar{\phi}_{mod}}{\bar{\phi}_{fuel}}$	1.2086	1.1388	1.090 ± 0.011
$\frac{\bar{\phi}_{fuel\ surface}}{\bar{\phi}_{fuel}}$	1.0759	1.0332	1.033 ± 0.015

TABLE 7.5

Values of the Thermal Utilization in the Enriched Lattice ($V_{mod}/V_{fuel} = 25.9$)

Experiment	0.9760 ± 0.0006
THERMOS	0.9738 ± 0.0006
ABH	0.9766 ± 0.0006

7.4 THE CAUSE OF THE DISCREPANCIES BETWEEN EXPERIMENTAL RESULTS AND THERMOS PREDICTIONS FOR THE ENRICHED LATTICE

A search for the discrepancies observed in the case of the enriched lattices was undertaken because of the usefulness and importance of the THERMOS code in the study of heterogeneous assemblies. Attention was focused on the assumptions on which THERMOS is based. The most vulnerable of these seemed to be:

1. The assumption of isotropic scattering (except for the option of making a $\bar{\mu}_o$ correction for the actual anisotropy of the scattering by D_2O).

2. The energy transfer kernel (originally the Brown-St. John kernel, but recently replaced by a new kernel developed by Honeck (3)).

3. The cylindricalization of the lattice cell (recently improved by the development by Honeck of a two-dimensional code).

Since the two-dimensional THERMOS code was not available when the first studies of the discrepancies were undertaken, calculations were made by Mr. R. Simms of this laboratory with the Honeck, Brown-St. John, and mass-2 kernels, and with and without the $\bar{\mu}_o$ correction for anisotropic scattering. Only very small changes were obtained in the theoretical neutron density distributions, indicating that the difficulty lay in the cylindricalization assumption. This assumption is, indeed, a serious one in the case of the enriched lattice for which the spacing between rods is only slightly more than one mean free path, in contrast to the natural uranium lattices in which the spacings were about five mean free paths. It has recently become possible, through the kindness of Dr. H. C. Honeck and the Brookhaven National Laboratory, to apply the new two-dimensional THERMOS code to the enriched lattice. Mr. Simms has obtained the preliminary results shown in Fig. 7.3, which indicate that the two-dimensional code predicts a neutron density distribution for gold activation which is much closer to experiment than does the one-dimensional THERMOS code. The modified one-dimensional calculation shown in Fig. 7.3 was obtained by replacing the usual cylindrical cell boundary condition by a special condition which approximates a hexagonal cell boundary more accurately. Details of this treatment will be published later by Honeck. The two-dimensional code requires much more machine time than the one-dimensional code, and efforts are under way to reduce the time required. It seems certain that the new code will remove the

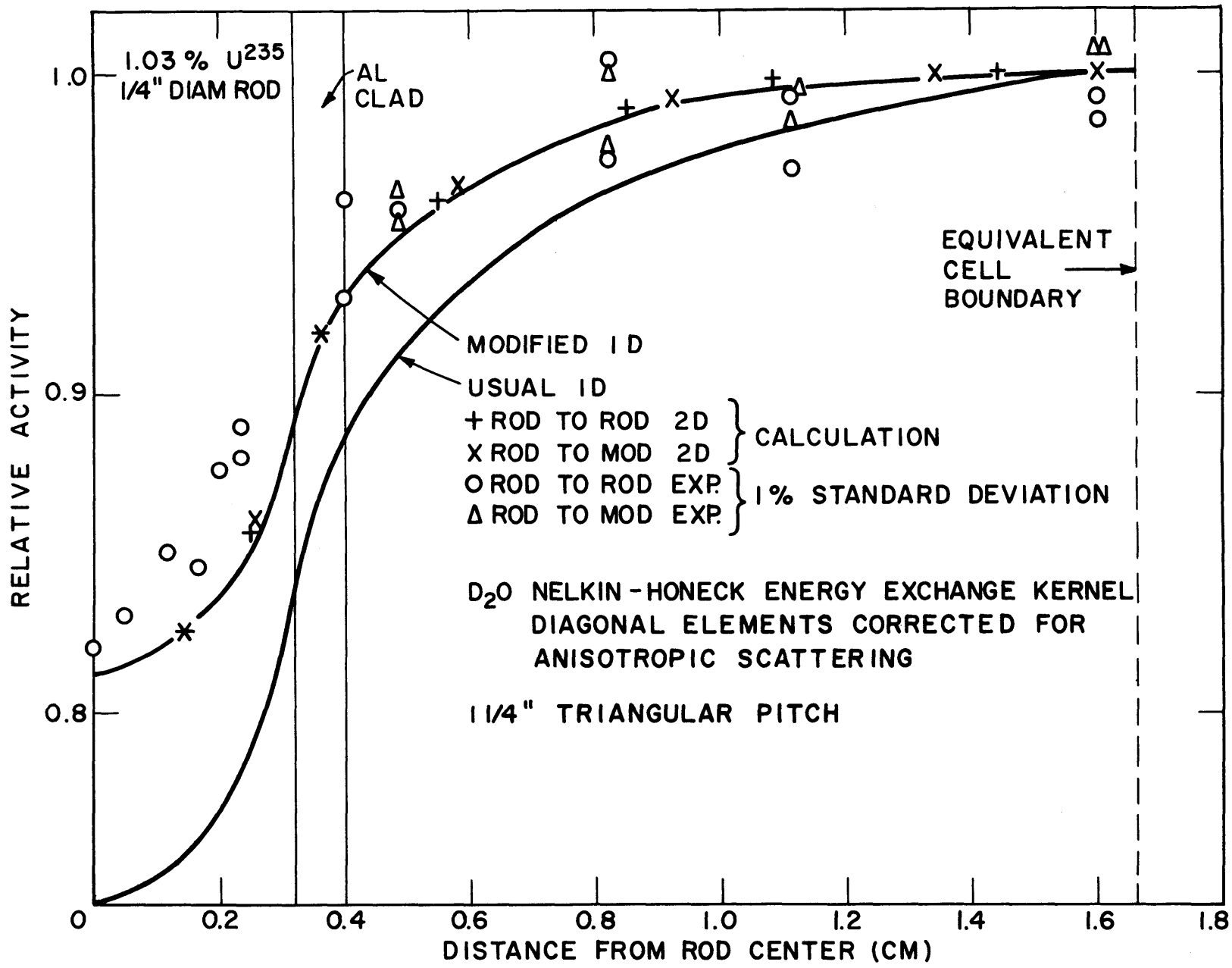

FIG. 7.3 PRELIMINARY COMPARISONS WITH 2D THERMOS CALCULATIONS OF RELATIVE GOLD ACTIVITY BELOW 0.41 ev

discrepancies and extend the usefulness of the THERMOS method to close lattices of thin, enriched rods.

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8. SMALL EXPONENTIAL ASSEMBLY RESEARCH

(J. C. Peak*)

8.1 LATTICES OF ENRICHED URANIUM RODS IN HEAVY WATER AND IN MIXTURES OF ORDINARY WATER AND HEAVY WATER: THEORETICAL RESULTS

The details of the methods and results of the work on small subcritical assemblies have been reported in NYO-10204 (cf. Introduction, ref. 5) by Peak et al. This work was undertaken because the use of small assemblies for the measurement of reactor parameters in the early study of new types of reactors offers potential savings in time and money. The results are summarized briefly here.

1. Age-diffusion theory was applied to the general case of a cylindrical subcritical assembly with a thermal neutron source at one end. The solutions allow calculation of the thermal neutron flux due to the source, the thermal neutron flux due to neutrons born and moderated in the assembly, and the slowing-down density at any particular value of the age, at any point in the assembly.

2. Methods of correction were developed for measurements of lattice parameters. They can be applied by using any valid representation of the thermal neutron flux and slowing-down density in the neighborhood of the experimental position. The corrections can be tested for consistency by applying them to experimental values obtained at different positions in the assembly. The most important correction is to the U^{238} cadmium ratio, which is related to the resonance escape probability.

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* Now at General Atomic, San Diego, California

3. The size of a subcritical assembly (in terms of its neutron behavior) may be characterized by the three dimensionless parameters: R/L, H/L, and τ_o/L^2, where R and H are the radius and height, respectively, of a cylindrical assembly, and L and τ_o are, respectively, the thermal neutron diffusion length and the age to thermal energies. A proposed experimental position in a subcritical assembly may be characterized by three quantities: the magnitude of the thermal neutron flux relative to the entering neutron current; the ratio of the lattice-born and moderated thermal neutron flux to the thermal neutron flux due to the source; the extent of the asymptotic region (defined as the region of constant cadmium ratio), if such a region exists. Tables of these three quantities have been prepared over a range of possible subcritical assemblies, for the case of the minimum-volume cylinder.

8.2 EXPERIMENTAL MEASUREMENTS

Measurements were made in nine small assemblies, 16 inches in height and 20 inches in diameter. Uranium rods, 0.25 inch in diameter, with a U^{235} concentration of 1.143%, served as fuel in lattices at spacings of 0.880, 1.128, and 1.340 inches, respectively. For each case, mixtures of heavy and ordinary water were used as moderator, at D_2O concentrations of 99.8, 90.3, and 80.2 mole per cent of D_2O, respectively. The following measurements listed in (a), (b) and (c) below were made in each assembly:

(a) Axial and radial flux traverses with bare and cadmium-covered gold foils;

(b) The U^{238} cadmium ratio in a fuel rod;

(c) The U^{238}/U^{235} fission ratio in a rod;

(d) Intracell flux traverses with bare and cadmium-covered gold foils were made in six of the nine assemblies.

8.3 EXPERIMENTAL RESULTS

(a) The experimental radial flux traverses agreed with those predicted by the theory in all nine lattices studied; the shapes of the curves were given correctly; quantitative agreement could be obtained by using the extrapolation distance, $1.0\ \lambda_{tr}$, instead of $0.71\ \lambda_{tr}$, where λ_{tr} is the thermal transport mean free path.

The experimental axial thermal neutron flux traverses agreed with the theoretical traverses provided extrapolation distances of $2.0\ \lambda_{tr}$ were used at the ends of the assembly. The need for the use of such extrapolation distances has been observed earlier, for example, in connection with exponential experiments in uranium-graphite assemblies. This type of problem is discussed at greater length in a review article to be published shortly as an NYO report. The theory also predicted the correct shape of the slowing-down density as a function of position along the axis of the assembly. Quantitative agreement between theory and experiment could be obtained by choosing the value of the effective resonance integral for the gold foils. Variations in this value, which is used merely for normalization purposes, simply shift the ordinates of the slowing-down curve up or down without changing the shape of the curve. Typical results are shown in Fig. 8.1 for one of the nine lattices.

(b) The results of the measurements of the U^{238} cadmium ratio and the calculated values of the ratio of source thermal neutron flux to the flux of lattice born and moderated neutrons showed that the lattices studied contained no asymptotic region. The values of the cadmium ratio varied with axial position; they could be corrected, by means of the theory developed, to give the values that should be obtained in measurements in a critical assembly. The results obtained in applying the corrections led to consistent results: for example, two values of $\rho_{28} = (R_{cd}-1)^{-1}$

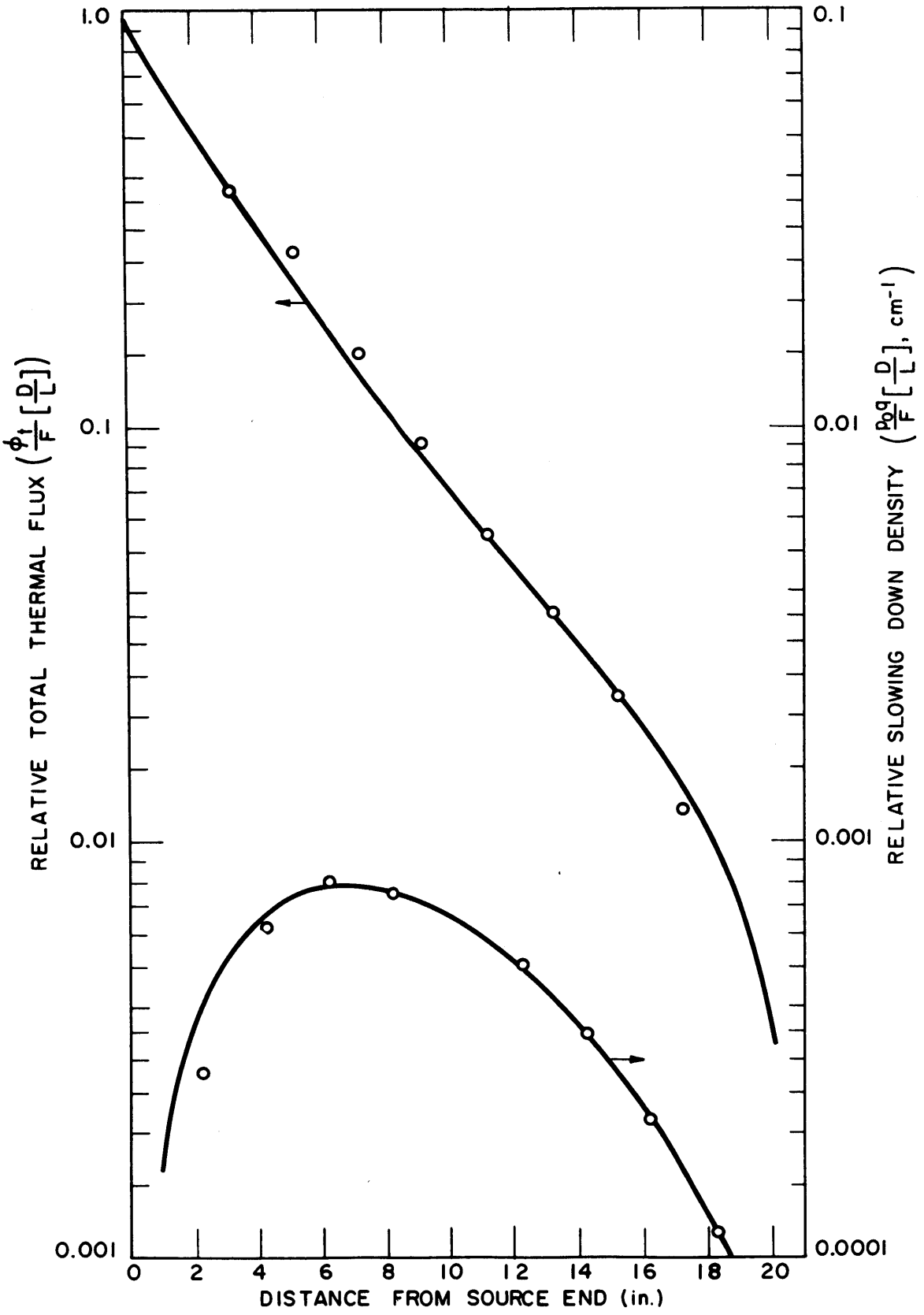

FIG. 8.1 AXIAL FLUX TRAVERSE (99.8% D_2O, 1.340 in. SPACING)

differing by as much as 30 to 40 per cent, gave corrected results differing by only 1 or 2 per cent. Although the corrections have been shown to lead to consistent results for the "critical assembly" values, it has not yet been shown that these consistent results are correct. They are to be compared with values measured in the large exponential assembly for which the results should be very close to the critical assembly values. Measurements are in progress, but it is too early to report any results.

(c) The values of δ_{28}, the U^{238}/U^{235} fission ratio, required relatively small corrections because of the small size of the lattice. As in the case of the cadmium ratios, the use of the corrections led to consistent results. Measurements are now being made in comparable lattices in the exponential assembly, to test the results obtained in the small subcritical assembly. Preliminary results indicate that the agreement should be satisfactory, but it is too early to quote results.

(d) Intracell Flux Traverses

The results of the measurements of intracell flux traverses provide an interesting complement to the results discussed in section 7 of this report. The flux traverses were measured in six lattices of 0.25-inch diameter rods, with a U^{235} concentration of 1.143%, at triangular spacings of 0.880 inch, 1.128 inches and 1.340 inches, respectively. The moderator, in one series of experiments, was water containing 90.3 mole per cent D_2O; in the second series, it contained 80.2 mole per cent D_2O. Unfortunately, a series of experiments with 99.8 mole per cent D_2O was spoiled because of experimental difficulties whose effects became known only after the D_2O had been diluted with ordinary water to the 90.3 mole per cent D_2O water; the experiments could, therefore, not be repeated. The flux traverses for the six lattices are shown in Figs. 8.2 to 8.7. The solid curves represent the results of theoretical distributions obtained with the THERMOS code mentioned in section 7

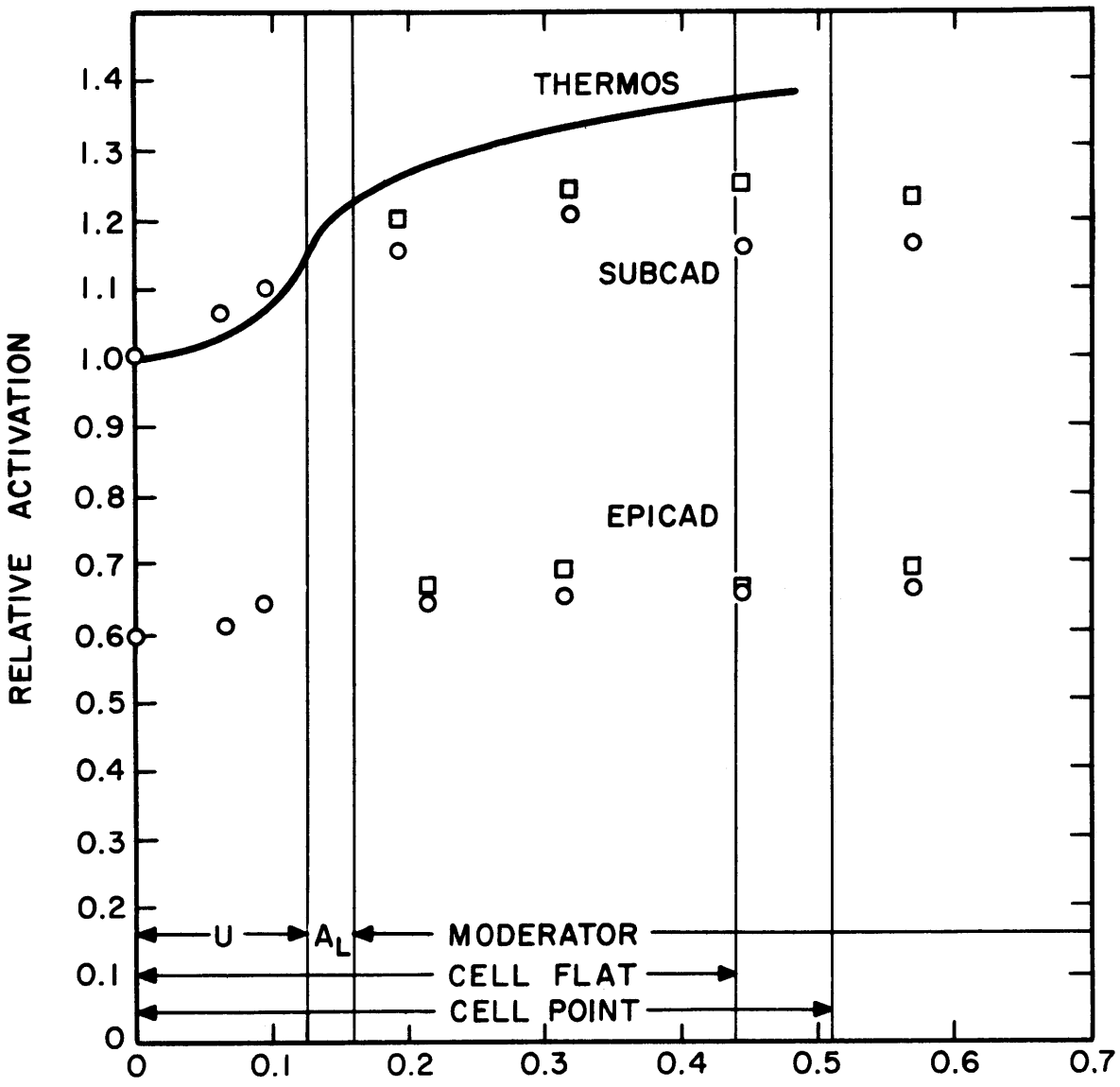

FIG 8.2 RADIAL DISTANCE FROM ROD CENTER (in.)

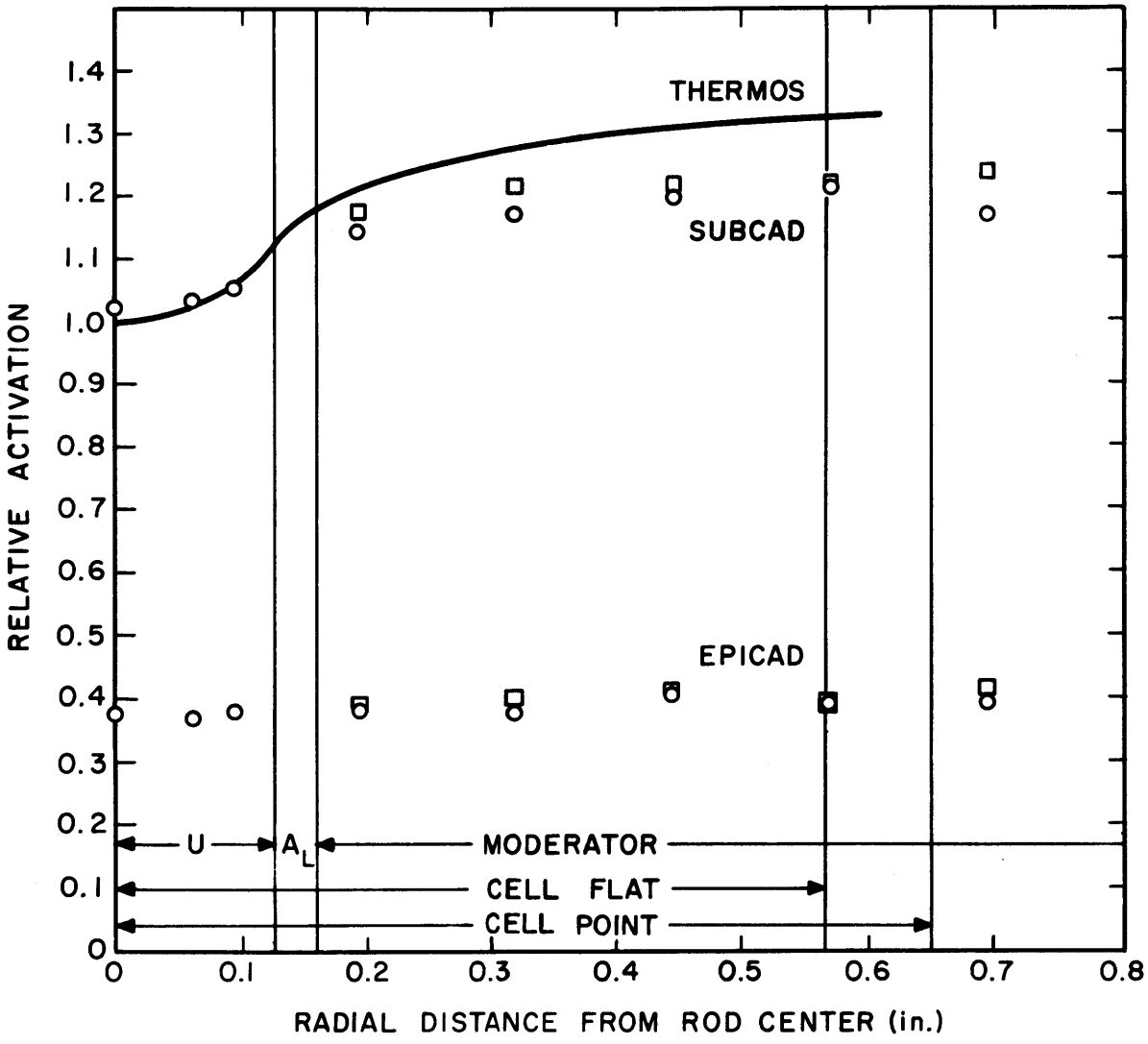

FIG. 8.3 INTRA-CELL GOLD FLUX TRAVERSE
(90.3% D_2O, 1.128 in. SPACING)

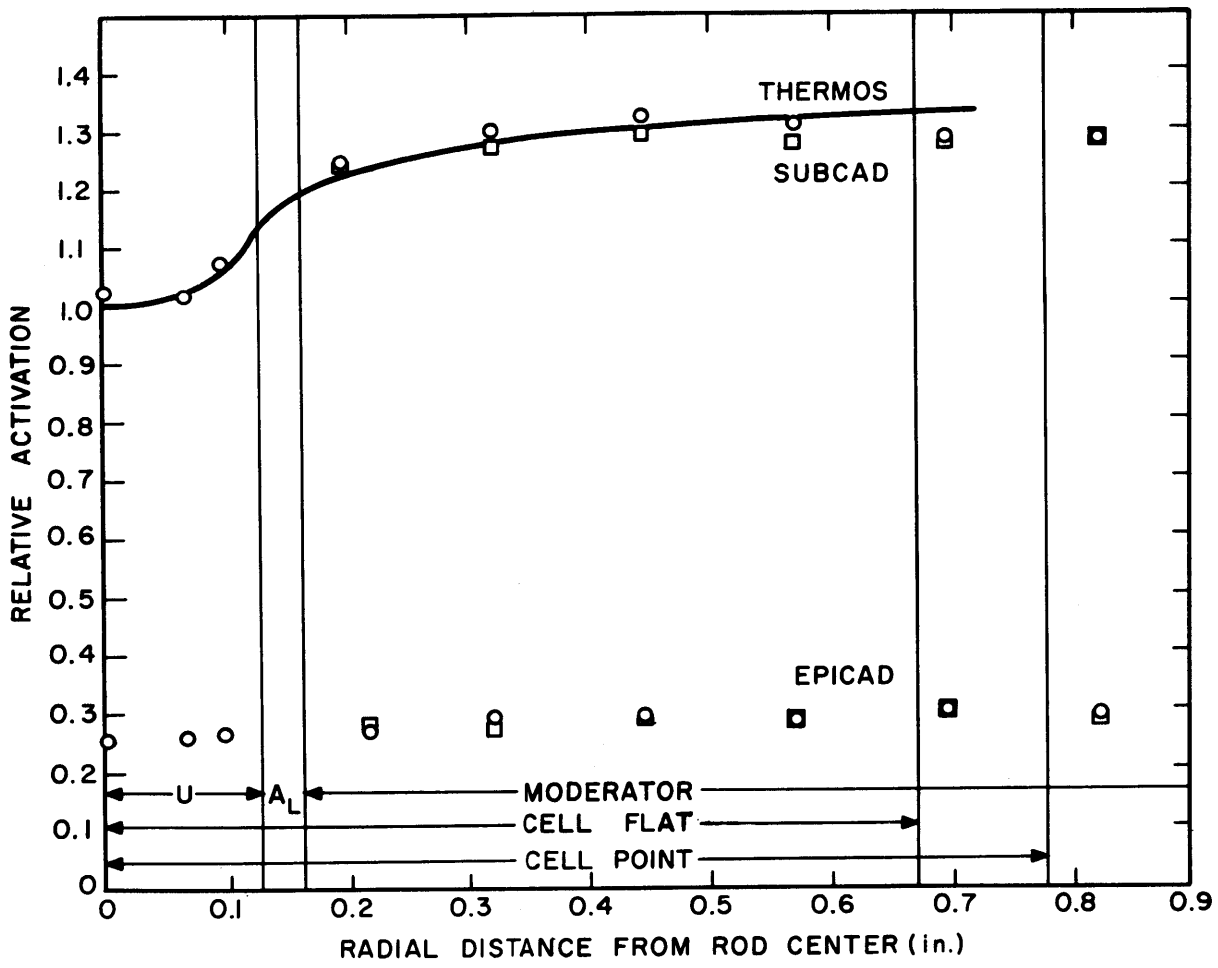

FIG. 8.4 INTRA-CELL GOLD FLUX TRAVERSE (90.3% D_2O, 1.340 in. SPACING)

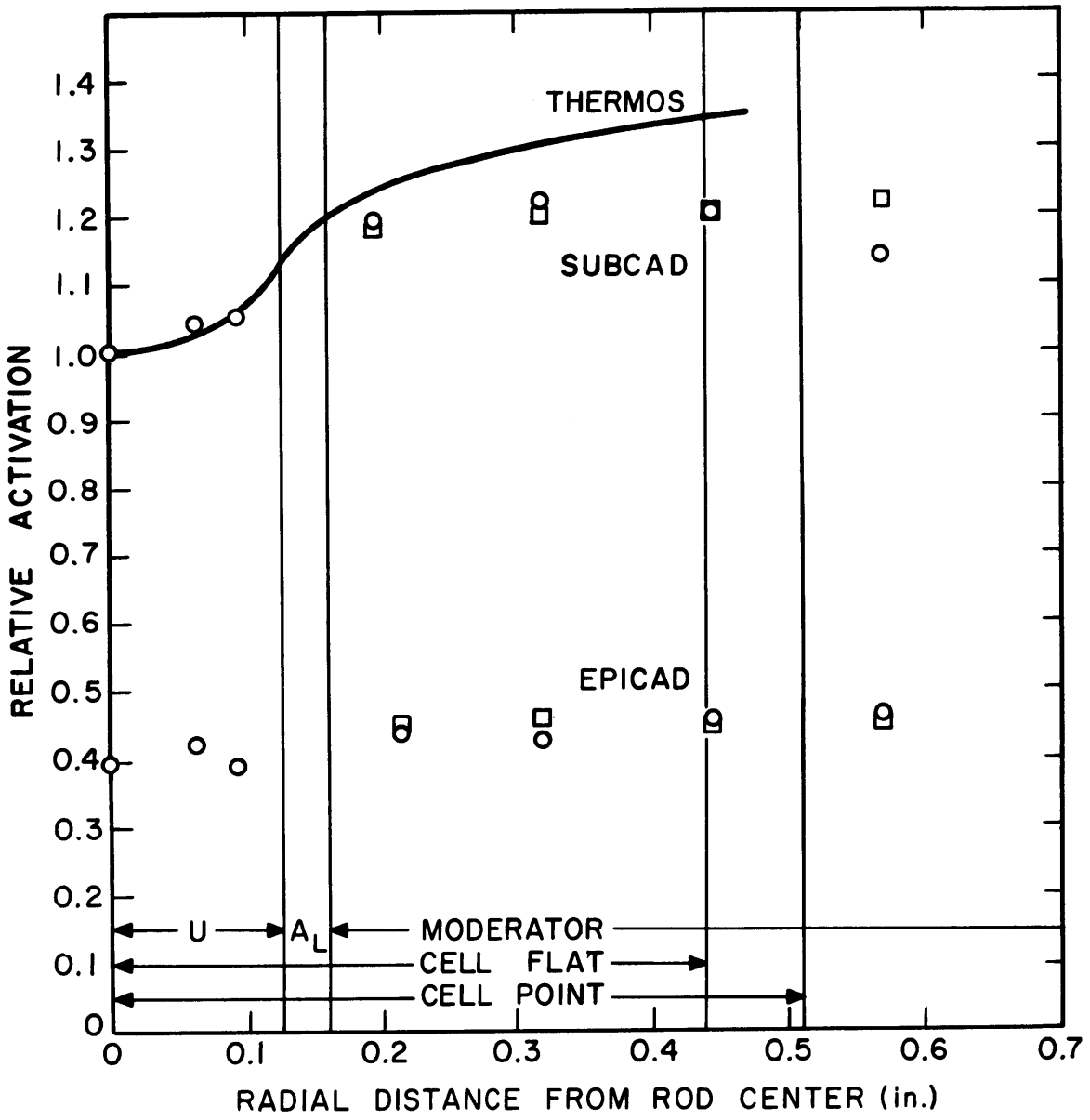

FIG. 8.5 INTRA-CELL GOLD FLUX TRAVERSE
(80.2% D_2O, 0.880 in. SPACING)

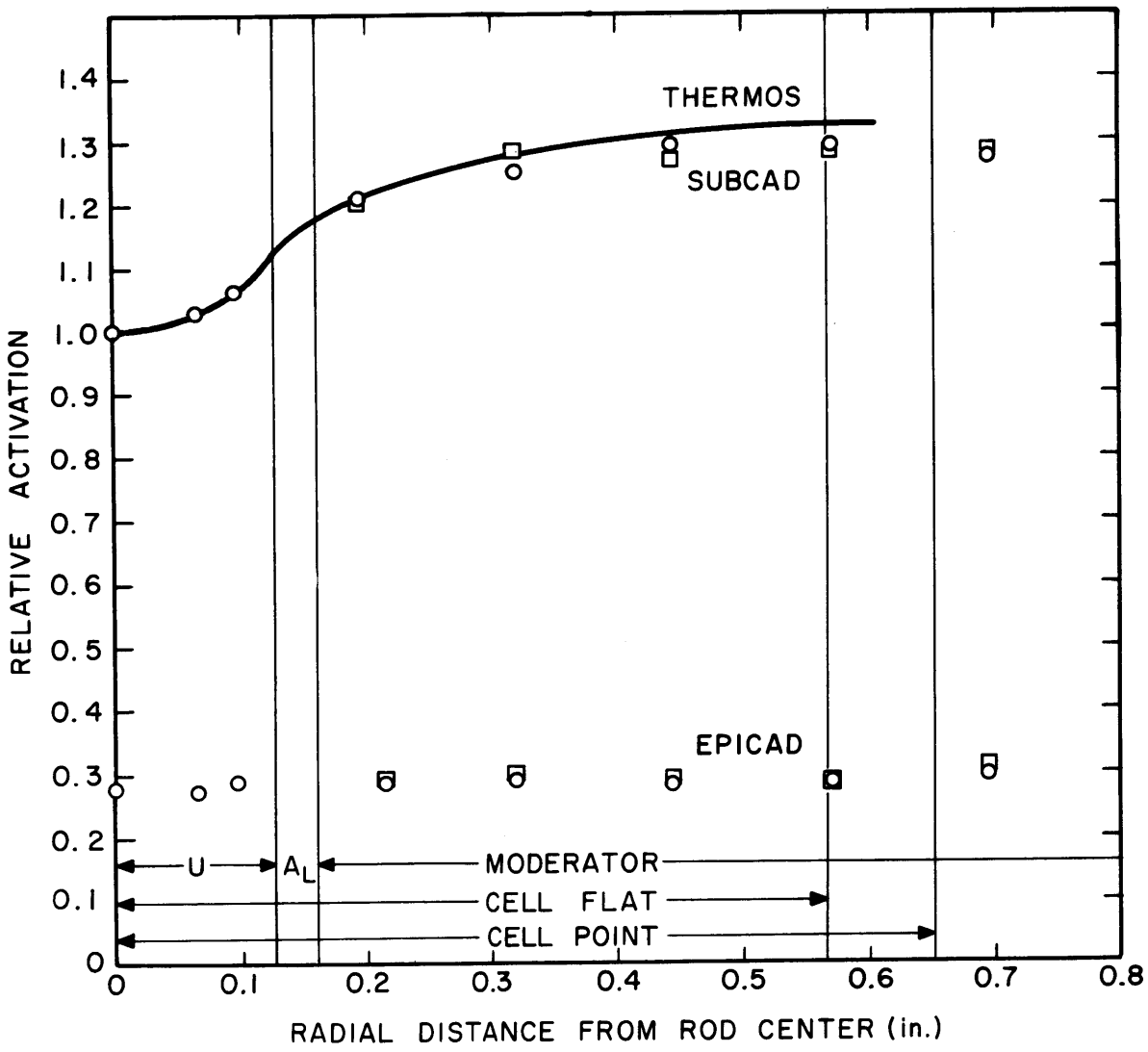

FIG. 8.6 INTRA-CELL GOLD FLUX TRAVERSE
(80.2% D_2O, 1.128 in. SPACING)

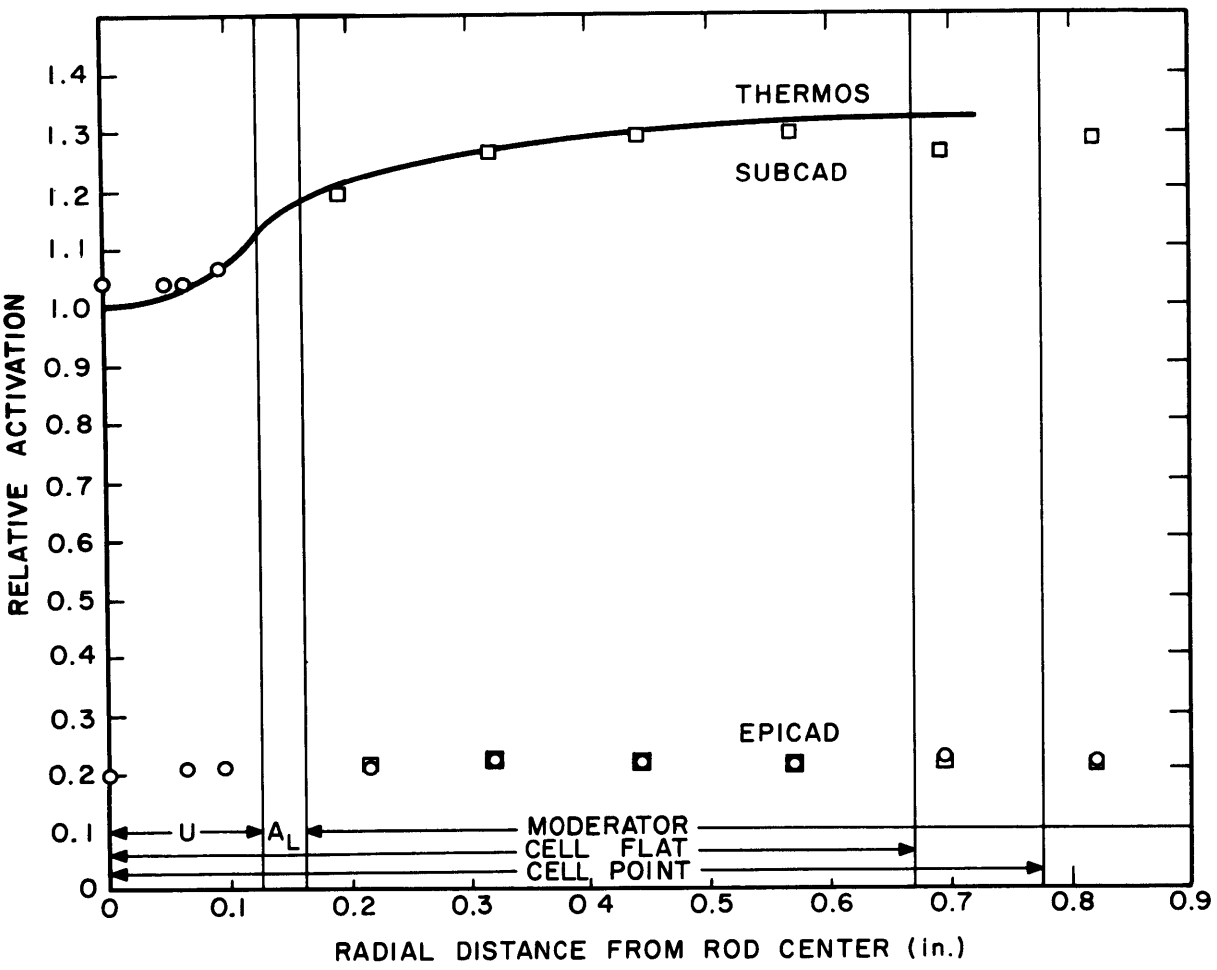

FIG. 8.7 INTRA-CELL GOLD FLUX TRAVERSE (80.2% D O, 1.340 in. SPACING)

of this report. Calculations were made with the THERMOS code modified to include the effect of leakage by introducing the term DB^2 into the cell calculations as an additional energy-dependent absorption term. Although this type of correction is far from rigorous, it should give some idea of the size of the correction needed in interpreting the results of measurements in the small lattices; more precise treatments are under consideration (cf. section 9 of this report). The theoretical effect, obtained by the rough method used, on a relative measurement of neutron density made with a 1/v-absorber such as gold turned out to be quite small; in all nine of the cases computed, the difference caused by the DB^2 term was never greater than 0.3 per cent. For this reason, the experimental intracell flux traverses are compared with the theoretical results for the infinite assembly to which THERMOS applies.

The results show that the agreement with the THERMOS theoretical predictions is good only in the lattices with the widest spacings. In the tighter lattices, there is a clear discrepancy between the theoretical and measured neutron densities. In view of the discussion of section 7, it seems probable that the major source of the discrepancy is the cylindricalization of the cell in the THERMOS calculations. The addition of H_2O increases the size of the cell (in terms of number of mean free paths) and tends to reduce the effect of the cylindricalization.

9. INTERPRETATION OF REACTOR PHYSICS MEASUREMENTS (R. Simms)

In connection with the experimental program of the lattice project, a theoretical program has been started. One report of a general nature, NYO-10206, by K. F. Hansen, has been published on the general formulation of multigroup methods (cf. Introduction, ref. 11). This section of the present report will deal with theoretical studies on the calculation of the quantities actually measured in the experiments. The emphasis, so far, has been on the calculation of the intracell thermal neutron activation rates measured with various detectors, gold, lutetium, and europium. The results of these measurements have been reported in detail in NYO-10205 by Brown et al. (cf. Introduction, ref. 6) and have been summarized in section 7 of the present report.

The interpretation of the measured intracell flux traverses and the calculations of disadvantages factors and thermal utilization from the foil activation distributions require several assumptions. One of the purposes of the present research is to modify the theoretical methods used so far, so that direct comparison between theory and experiment will be possible without these intermediate assumptions and corrections. In particular, the attempt will be made to calculate the neutron energy spectrum as a function of position, taking into account the finite size of the assembly and investigating the effects of various assumptions used in the theory.

The theory involves the calculation of space-dependent neutron spectra, which presents a highly difficult and complicated problem if realistic energy exchange kernels are used. Various computer codes have been developed to treat the problem, differing in the transport approximation made; Monte Carlo calculations have also been used.

Probably the most rapid method developed to date for computing space-dependent spectra for lattice cells is the SWAKRAUM code (1). By using a non-self adjoint variational method, the P-1 multigroup equations are reduced to a few coupled Helmholz equations, the number depending on the number of trial spectra chosen. The space-dependent spectrum is taken as a linear combination of the trial spectra, the coefficients being determined by optimizing the absorption rate. The method also includes an option for the DP-1 or P-3 approximations in slabs. The variational method has been used by Buslik (2) for the self-adjoint equations.

The basic weakness with the variational method is in the selection of the trial spectra. It has been shown that the choice of trial spectra can lead to uncertainties as large as the differences obtained between the use of the mass one and Nelkin kernels for water-enriched uranium systems (3).

An alternative to the variational method involves the direct, iterative solution of the multigroup P-1 equations. ULCER (4) and SLOP-1 (5) are good examples of the use of diffusion theory. The computer time required becomes substantially longer than for the variational methods, but the uncertainty inherent in the choice of trial spectra no longer exists. The SLOP-1 code has an option for DP-1 or P_3 treatments in slab configuration, with the scattering treatment to the order of P_3 scattering terms, but with energy transfer allowed only in the P_o component.

For cylinders, these codes suffer from the defects inherent in the use of diffusion theory, and there seems to be little incentive to code a multigroup version for cylinders based on a higher-order spherical harmonic approximation.

The T. E. T. code (6) offers the desired degree of transport approximation for the slab configuration. The code is a form of the discrete ordinate method in that it uses six quadrature angles in the forward and backward directions. This order of approximation would be equivalent to a DP-5 approximation. Eventually, the code will incorporate terms up to the P_3 scattering term, with

P_3 source terms. It is not likely to be extended to cylinders.

The S_N method (7) has received much attention in recent years. What the method lacks in elegance, it makes up in adaptability to computing machines. A British report indicates that their version of the SNG code, in S_8, for cylinder has been used with good results (8).

The integral transport equation has been coded by H. C. Honeck (9) for both slab and cylindrical configurations. The basic difficulty in this code, THERMOS, lies in its assumption of isotropic scattering. For the slab, this restriction has been relaxed and P_1 scattering considered (10). However, the inclusion of P_1 scattering involves considerable effort, and little incentive exists for including P_1 scattering for the cylindrical version.

The Monte Carlo approach offers the most exact method, the degree of exactness depending on the amount of computer time one is willing to spend. TRAM (11) and MARK (12) are examples of the Monte Carlo treatment.

In the work of the M. I. T. Lattice Project, computer time is limited and it was decided to select a method that is reasonably exact but not as time-consuming as Monte Carlo. The choice was THERMOS, because of its proven usefulness and the possibility of close cooperation with the Reactor Theory Group of the Brookhaven National Laboratory, where Dr. Honeck is developing the code further. It would also be desirable to use the DSN code.

Calculations have been made, mostly at the M. I. T. Computation Center, for the cells investigated at the M. I. T. Heavy Water Facility. The results of the calculations for three scattering kernels and two types of anisotropic scattering corrections are compared in Table 9.1 for 1.0-inch diameter, natural uranium rods on a 5.75-inch triangular spacing, in heavy water.

The simplest scattering model for D_2O would be to assume a free gas with the proper mixture of mass two and mass 16 atoms. Since the atoms are bound in the molecule, the energy transfers

TABLE 9.1

Relative Activation of a 1/v-Absorber at Energies Below 0.4 Ev for One-Inch, Natural Uranium Rod On a 5.75-Inch Triangular Spacing, in D_2O

		Model*				
	Radius, cm	A	B	C	D	E
Fuel	0.000	1.0000	1.0000	1.0000	1.0000	1.0000
	0.284	1.0188	1.0186	1.0185	1.0183	1.0185
	0.568	1.0684	1.0675	1.0670	1.0666	1.0671
	0.853	1.1634	1.1613	1.1601	1.1594	1.1604
	1.137	1.3374	1.3330	1.3303	1.3290	1.3309
Clad	1.324	1.5285	1.5212	1.5164	1.5147	1.5177
Mod	1.762	1.7707	1.7543	1.7478	1.7342	1.7433
	2.549	1.9804	1.9554	1.9503	1.9196	1.9362
	3.336	2.1208	2.0905	2.0873	2.0440	2.0658
	4.123	2.1899	2.1568	2.1544	2.1048	2.1292
	4.910	2.2494	2.2141	2.2133	2.1569	2.1834
	5.697	2.2943	2.2578	2.2575	2.1975	2.2253
	6.484	2.3253	2.2880	2.2881	2.2264	2.2547
	7.272	2.3459	2.3090	2.3087	2.2481	2.2761

*Description of the models used:
A = Mass two
B = Brown-St. John
C = Nelkin-Honeck
D = $\bar{\mu}$ correction to C
E = Diagonal correction to C

would be too large and the moderation would be overestimated. Brown and St. John (13) attempted to correct for the effect of chemical binding by considering the D_2O molecule to be a rigid rotator. They then derived an effective mass for the deuteron, 3.595 amu, to be used in the free gas model. Honeck (14) has extended the treatment of Nelkin (15) to D_2O. Nelkin assumes that the hydrogen in H_2O scatters incoherently, and develops the differential scattering cross section, $\frac{\partial^2(E\rightarrow E',\mu)}{\partial E \partial \mu}$, from basic data on rotational and vibrational measurements. Honeck has done the same thing for D_2O; the incoherent approximation may not, however, be as good for D_2O as H_2O.

Since THERMOS takes into account only isotropic scattering, an attempt has been made to approximate the effect of the anisotropic scattering by:

(1) Multiplying $\Sigma(E\rightarrow E')$ by $1-\bar{\mu}$, where $\bar{\mu} = 2/3\ A_{eff}$, and A_{eff} is the Brown and St. John effective mass;

(2) Using $\Sigma_s^*(E\rightarrow E') = \Sigma_s(E\rightarrow E') - \left[\int dE'\ \Sigma_{s1}(E\rightarrow E')/\Delta E \right]$ in the Nelkin-Honeck model. It is noted that only the diagonal terms are affected in (2); $\Sigma_{s1}(E\rightarrow E')$ is the first Legendre component of the differential energy exchange cross section, $\Sigma_s(E\rightarrow E')$ being the zeroth.

The results indicate that the relative activation rates depend only slightly on the energy exchange kernel and somewhat more on the anisotropic approximation used. The comparison with the experiment is reported in NYO-10205, and an example is shown in Fig. 7.1 for 1.0-inch diameter rods, with a lattice spacing of 5.75 inches in D_2O.

Work is in progress on extension of the THERMOS code to include the effects of the finite size of the lattice, in both axial and radial directions. The two-dimensional THERMOS code developed by Honeck has also been used to account for discrepancies

between theory and experiment observed in tight lattices of 0.25-inch diameter rods (cf. section 7 of this report).

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10. PULSED NEUTRON RESEARCH

(A. E. Profio)

10.1 THE PULSED NEUTRON SOURCE

The Texas Nuclear Corporation pulsed neutron source was described in the first annual report, NYO-9658, although not supplied as part of this contract. It is a 150-kv Cockroft-Walton accelerator which may be operated with either a titanium deuteride or a titanium tritide target. With the former target, about 5×10^8 neutrons/sec are produced during the pulse, at an average energy of 2.5 Mev. With the tritium target, up to 10^{11} neutrons/sec are produced, at an average energy of 14 Mev. The accelerator is equipped with a versatile beam deflection pulsing system. Pulse lengths from 10^{-8} sec to more than 10^{-3} sec may be selected, at repetition rates from 10 to 10^6 pps. The accelerator can also be pulsed in a single shot or repetitively at less than 10 pps if driven by an external pulse generator.

The accelerator was installed by the manufacturer in July, 1961. For the next few months, it was operated with a dummy target while various tests and modifications were made. A 60-inch long drift tube was added to extend the target into a shielded room. An electrostatic quadrupole lens made by the Texas Nuclear Corporation was installed between the target and drift tube. This lens improved the focussing of the beam and permitted a threefold increase in target current. The shape of the pulse was not entirely satisfactory at first; it was improved by installing a new terminal square wave generator provided by the manufacturer. Later, after the neutron producing target was installed, it was found that too much beam current leaked past the terminal deflection plates when the beam was supposed to be cut off. The background neutron

production between pulses was too high and the flux decay could not be followed over several decades as desired. A second set of deflection plates, located after the accelerating column and pulsed in synchronization with the terminal deflection plates, solved this problem.

10.2 MEASUREMENTS

Many measurements of thermal dieaway time constants have been made, mostly on toluene, as part of another investigation. The time analyzer was a Technical Measurements Corporation 256 channel pulse analyzer with the Model 212 "Pulsed Neutron" plug-in logic unit. The analysis cycle begins with a trigger from the repetition rate generator. The analyzer takes a background count, triggers the accelerator, delays for a preset time, and then accumulates counts sequentially in time intervals adjustable between 10 μsec and 2560 μsec.

Many other measurements have been made of counting rate versus time distribution in the few-microsecond range, in an attempt to measure slowing-down times. The Technical Measurements Corporation Model 211 "Times of Flight" plug-in logic, or a nanosecond time-to-amplitude converter and pulse height analyzer were used in these investigations. Time-gated pulse height spectra were taken to measure absorptions at various intervals during and after the burst, by observation of the prompt gamma radiation. The absorptions due to fast, resonance, or thermal neutrons should be separable by time discrimination. The slowing-down time versus energy scale can be established experimentally by investigating known resonance and thermal non-1/v absorbers.

The investigations mentioned above were supported by another agency; a complete account will be published elsewhere. However, invaluable experience was gained and the accelerator and pulse analyzing equipment are now prepared for research on the lattice program.

10.3 PROPOSED RESEARCH

It is proposed to concentrate on neutron thermalization in future research with the Texas Nuclear accelerator. Thermalization is the process in which the moderated neutron exchange energy, with the moderator nuclei as an asymptotic spectrum, is approached. The energy exchange is dependent on the structure and thermal motion of the moderator molecules. A study of the asymptotic spectrum as a function of chemical binding, (material), temperature, amount of 1/v absorption, and size is under way at General Atomic with the pulsed source, time-of-flight method pioneered by Poole in England. Recently, some graphite measurements have been made by Poole and colleagues in which a pulsed source and a chopper were used to measure the spectrum as a function of time after the burst. Time-of-flight research of this type is beyond the capability of the pulsed sources available to us. Spectra can be measured with choppers, with the M. I. T. research reactor as the steady source. The pulsed source, time-of-flight method is restricted by the requirement for a reasonably short dieaway time, and a chopper is probably better for lightly poisoned D_2O-moderated assemblies.

Another approach to the study of thermalization is the measurement of the neutron density dieaway time constant following injection of a neutron burst. The asymptotic spectrum is found to be a function of the size of the block, and the time for the neutron density to die away is affected because the diffusion coefficient, and hence the leakage rate, is different. In most moderators, the absorption cross section and the transport mean-free path vary in such a way that the spectrum is cooled. That is, the asymptotic spectrum has a lower average energy than the Maxwellian at the moderator temperature and the leakage is smaller than would be found if the spectrum were unchanged. The dieaway time constant may be written as a series in the buckling B^2:

$$\lambda = v\Sigma_a + vD_oB^2 - CB^4 + \ldots ,$$

where Σ_a is the macroscopic absorption cross section, vD_o is the diffusion coefficient averaged over the Maxwellian at the moderator temperature, and C is the diffusion cooling coefficient. By measuring λ as a function of B^2, the various coefficients may be determined. A few measurements (1, 2) have been made in D_2O, but previous investigators have worked with very small volumes ($B^2 > 0.06$ cm^{-2}). At M. I. T., relatively large quantities of 99.8% D_2O and various sized tanks are available, as well as the pulsed sources and other equipment needed for measurements over a wide range of bucklings.

The diffusion cooling coefficient is related by various approximate theories to the mean-square energy transfer cross section. A different way to arrive at this quantity is to measure the higher time modes, which represent the way in which the asymptotic spectrum is approached. The lowest mode in an infinite medium without absorption decays with the thermalization time constant. One way to measure the time constant is to observe the capture gamma-ray counting rate versus time from a non-1/v absorber such as cadmium dissolved in the moderator. The amount of absorber is made small enough so that the spectrum is not appreciably perturbed. Such measurements have been made by Möller and Sjöstrand (3) in a large volume of water. A small scintillation detector was used to observe the gamma rays. In such measurements, care must be taken to account for the effect of local differential leakage in the measured time constant, and to subtract the background due to captures in the moderator. Similar measurements have been made at M. I. T. with the Texas Nuclear accelerator, with the volume integration being made by a large liquid scintillation counter. It is proposed to continue the investigations of the thermalization time constant, to see if the technique can be applied to D_2O.

There are several other items which ought to be investigated, e. g., the variation in spectrum with distance from the source or a boundary, and the variation of the vacuum boundary extrapolation distance with buckling (4, 5). The chopper or a crystal spectrometer

may be used for the former. A miniature traveling neutron detector may be used to measure pulsed extrapolation distances (6). Only a small amount of theoretical work and only preliminary experiments have been done in these areas.

10.4 A SECOND PULSED NEUTRON SOURCE – THE K TUBE

In the latter part of the year, it became obvious that much more work along the present lines remained to be done with the Texas Nuclear accelerator. The possibility arose of obtaining a small, portable, sealed discharge type neutron source for the lattice experiments. Recently, there has been developed a new pulsed source of this type, the K tube. The K tube uses a Penning cold cathode ion source and a tritiated target which is pulsed at −150 kv during the neutron burst. There is no vacuum pump, but the ion source and accelerating gap operate at a deuterium pressure of about 10 microns. The pressure is measured by a Pirani gauge and controlled by a heater on a titanium deuteride dispenser. The neutron output is 10^8 neutrons per 15 μsec burst and the maximum recommended repetition rate is 1 to 10 pps, the higher rate resulting in a shorter lifetime. The large pulse output, low repetition rate generator is well suited for the large, D_2O-moderated subcritical lattices. A K tube has been ordered and the construction of the HV modulator is well along. We are indebted to Dr. David Bach at the Knolls Atomic Power Laboratory for information concerning his work with the K source and for sending schematics of the modulator.

The detector for the lattice experiments will be a bank of N. Wood enriched BF_3 proportional counters. A Radiation Instrument Development Laboratory amplifier and HV supply will be used. The time analyzer will be the TMC Model 212 unit in the 256 channel analyzer. Since the lattice pulsed source experiments, the Texas Nuclear pulsed source experiments, the proposed chopper experiments, and the pulse height analysis of fission products and other lattice parameter measurements all use a multichannel analyzer,

one was obtained for the exclusive use of the lattice project.

The K source will be used to measure the D_2O absorption cross section and diffusion coefficient as a check on the heavy water purity. These measurements will extend the thermalization measurements to the large bucklings afforded by the exponential tank. The source can also be used at the startup of each new lattice to be certain it is safely subcritical. However, the most important use will be to measure reactivity coefficients in the slightly enriched uranium lattices which are being studied under this project. The pulsed source will also be an essential tool in the control rod research discussed in another section of this report.

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11. CONTROL RESEARCH

(B. K. Malaviya)

The program of research on the physics of the control aspects of thermal reactors, under the Lattice Project, falls into three broad categories:

I. Exponential Experiments with Control Rods
II. Reactivity Studies with Pulsed Neutron Techniques: Theoretical and Experimental Investigations
III. Control Rod Theory for Improved Analytical Methods of Calculating Control Rod Worths.

These three phases of the control research are, of course, related and provide intercomparisons and counterchecks leading to a better understanding of the lattice physics involved.

11.1 EXPONENTIAL EXPERIMENTS WITH CONTROL RODS

Although control rod studies are usually undertaken in conjunction with a critical facility, an assembly of the PCTR type, a pulsed subcritical assembly, or an operating reactor, exponential experiments can provide interesting results in certain cases. The type and scope of such experiments is limited, and easily interpretable results are possible only in cases where single control rods are fully inserted parallel to the axis of relaxation of the exponential assembly. For a cylindrical tank (extrapolated radius R) of pure moderator with a thermally-black rod (radius a) along the axis, one-group analysis yields the following expression for the thermal extrapolation distance into the rod:[1]

$$\frac{1}{d_2} = \frac{1}{a} \frac{Y_o(aR)J_o(aa) - Y_o(aa)J_o(aR)}{Y_1(aa)I_o(aR) - Y_o(aR)J_1(aa)} , \qquad (11.1)$$

where α is the radial buckling related to the material buckling B_m^2 and the inverse relaxation length γ, by:

$$B_m^2 = \alpha^2 - \gamma^2 \ . \tag{11.2}$$

The radial buckling, α^2, can be determined from measurements at the axial bucklings γ_o^2 (without the rod), and γ^2 (with the rod). Since the material buckling (for the moderator region) remains unchanged,

$$B_m^2 = \alpha_o^2 - \gamma_o^2 = \left(\frac{2.405}{R}\right)^2 - \gamma_o^2 \ . \tag{11.3}$$

From Eqs. 11.2 and 11.3, we get α^2, and Eq. 11.1 can then be used to determine the extrapolation length d_2 into the rod of radius a.

Once d_2 is determined, Eq. 11.1 can be used to transform the result from an exponential tank to a full-sized bare reactor. If it is assumed that the effect of the outer boundary transients on the center of the reactor is negligible, so that d_2 remains unchanged, then, for a full-sized reactor of core-radius R_o, we must have

$$\frac{1}{d_2} = \frac{1}{\alpha_o} \frac{Y_o(\alpha_o R_o)J_o(\alpha_o a) - Y_o(\alpha_o a)J_o(\alpha_o R_o)}{Y_1(\alpha_o a)J_o(\alpha_o R_o) - Y_o(\alpha_o R_o)J_1(\alpha_o a)} \ ; \tag{11.4}$$

α_o^2 is now the radial geometrical buckling of the actual reactor with the rod axially inserted and can be calculated by solving Eq. 11.4 graphically or by iteration. For the reactor without the rod, α is $\left(\frac{2.405}{R_o}\right)$ so that the change in radial buckling for the reactor due to rod insertion is

$$(\Delta\alpha^2)_{\text{reactor}} = \left(\frac{2.405}{R_o}\right)^2 - \alpha_o^2 \ . \tag{11.5}$$

The axial buckling in each case is $\gamma^2 = \left(\frac{\pi}{H}\right)^2$, so that Eq. 11.5 also gives the change in the geometrical buckling of the reactor due to rod insertion:

$$(\Delta B_3^2)_{\text{reactor}} = \left(\frac{2.405}{R_o}\right)^2 - \alpha_o^2 \ . \tag{11.6}$$

Actually, we can eliminate d_2 between Eqs. 11.1 and 11.4 and then the result, Eq. 11.6, is insensitive to the precision with which d_2 is determined.

If now we carry out a two-group analysis of a multigroup exponential assembly with a fully inserted thermally black control rod along its axis, we are led to the following expression:[1, 2]

$$S_1 \frac{\frac{x_1}{x_1'} - d_2}{\frac{x_1}{x_1'} - d_1} = S_2 \frac{\frac{x_2}{x_2'} - d_2}{\frac{x_2}{x_2'} - d_1}, \tag{11.7}$$

where

$$x_1 = J_o(a_1 a) - \frac{J_o(a_1 a)}{Y_o(a_1 R)} Y_o(a_1 a),$$

$$x_2 = I_o(a_2 a) - \frac{I_o(a_2 R)}{K_o(a_2 R)} K_o(a_2 a).$$

Here, R is the extrapolated radius of the core (assumed the same for the fast and thermal groups),

$$a_1^2 = \mu^2 + \gamma^2,$$

$$a_2^2 = -\nu^2 + \gamma^2,$$

γ is the inverse relaxation length, μ^2 and $-\nu^2$ are the roots of the two-group critical equation

$$(1+L_1^2 B^2)(1+\tau B^2) = k_\infty,$$

S_1, S_2 are the fast and thermal coupling coefficient for fluxes in multiplying media, such that

$$\frac{S_1}{S_2} = \frac{1 - L^2\nu^2}{1 + L^2\mu^2},$$

and primes denote differentiation with respect to a.

Equation 11.7 can be used in several ways. First, if d_2 has been determined from measurements in pure moderator, the other parameters, except d_1, can be calculated for the lattice, and a measurement of the axial buckling γ^2 will yield the fast extrapolation distance into the rod in question. Further, if the rod is "transparent" to fast neutrons, e.g., a hollow air-filled control rod, then $d_1 \to \infty$, and we have

$$\frac{S_1}{S_2} = \frac{\frac{x_2}{x_2'} - d_2}{\frac{x_1}{x_1'} - d_2} . \tag{11.8}$$

We can also measure the change in the "major" radial buckling" δa_1^2 due to the insertion of the rod in the lattice. For this, we consider:

$$a_{10}^2 = \mu^2 + \gamma_o^2 \qquad \text{without the rod,}$$

$$a_1^2 = \mu^2 + \gamma^2 \qquad \text{with the rod in,}$$

whence

$$\delta a_1^2 = \gamma^2 - \gamma_o^2 = \delta\gamma^2 .$$

An extension of this result to a full-size reactor is similar to the case earlier considered.

The experiments currently under way involve the measurement of the thermal extrapolation distance d_2 for black cylinders of varying radius. Inasmuch as d_2 is an important input parameter in the evaluation of the control rod effectiveness, the results of these measurements will supplement control rod studies by other methods. But, more importantly, the calculation of d_2 is in itself a major theoretical problem in neutron transport theory: while theory has provided asymptotic expressions for d_2 in the limiting cases of $a < 0.15\,\lambda_{tr}$ and $a > 4\,\lambda_{tr}$, no definite results

are available in the intermediate range which is of more practical interest (3). A systematic set of measurements is planned to fill this gap.

The current experiments are being performed in the 48-inch diameter exponential tank of the MITR subcritical facility. The control rods have been constructed by wrapping two layers of 20-mil cadmium tightly around a 6061 aluminum buckling cylinder and soldering off the ends. This procedure yields a control rod having the cross section shown in Fig. 11.1. The rods are hollow inside and are closed at the bottom end by an aluminum disc which fits in the lower rod positioner. Thus the rods are black to thermal neutrons and almost transparent to fast neutrons – providing well-defined boundary conditions at the rod-moderator interface. The rod is firmly fixed along the axis by top and bottom positioners. The flux is mapped in the axial direction by 0.25" × 0.005" gold foils attached to 6061 aluminum foil holders.

Control rods of other types, e.g., B_4C-filled in aluminum tubes and moderator-filled cadmium tubes, are also under construction; these are partially opaque to fast neutrons and are expected to provide a more diversified set of experiments involving the fast extrapolation length and change of criticality due to rod insertion in a lattice.

11.2 REACTIVITY STUDIES WITH PULSED NEUTRON TECHNIQUES

The main part of the control rod work is concerned with the development of the pulsed neutron technique in relation to a multiplying subcritical assembly and its application to the measurement of the reactivity worth of lumped neutron absorbers in subcritical assemblies. Although the pulsed neutron method is already a well-established tool for the study of problems of thermal neutron diffusion (4, 5), its application in the measurement of reactivity is still in its early stages and relatively little work has been reported(6, 7).

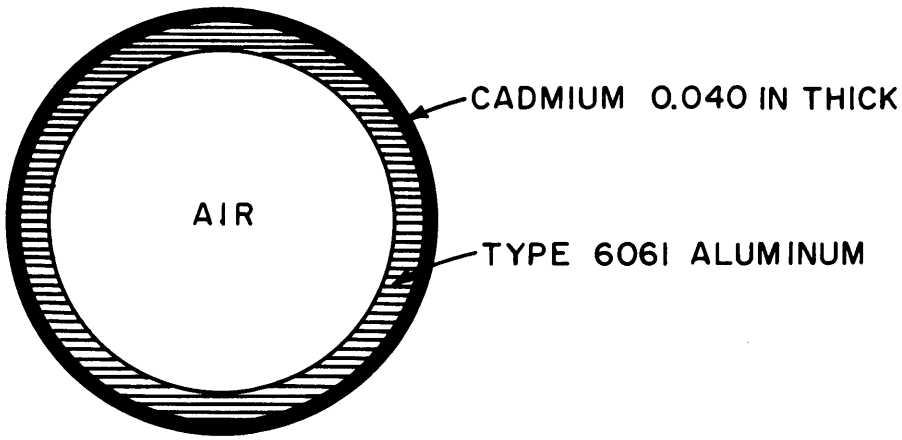

FIG. II.1 CROSS-SECTIONAL VIEW OF A THERMALLY BLACK CONTROL ROD

Theoretical studies have been made in connection with the planning of the experiments and the interpretation of the results. Thus, if the source is at a height equal to H/2, the even axial harmonics are automatically eliminated (not excited). Furthermore, a calculation of the decay constant of the various modes for several typical lattices of enriched uranium in heavy water has shown that the decay constants of the higher modes are greater than the decay constant of the fundamental by at least a factor of two in each case, so that even a short waiting time for the stabilization of the thermal flux will result in a fairly complete isolation of the fundamental mode.

The initial pulsed neutron work preparatory to pulsing the subcritical assemblies has been done with the 150-kv Texas Nuclear Corporation accelerator discussed in Section 10 of this report. Some "die-away" experiments on simple moderators have been undertaken. These experiments were intended to check out the various components and the over-all circuitry and to gain experience in the manipulation of the source and detector locations. The instrumentation has been formalized and adapted; and we have tried several means for suppressing the higher spatial harmonics, using various BF_3 counters and Li^6 loaded ZnS and plastic fluor mounted on a Dumont 6364 (5-inch) photomultiplier tube. Following a suggestion from A. E. Profio, it is planned to use long BF_3 counters, stacked longitudinally along the lateral periphery of the experimental tank, with the height and position adjusted in such a way that the unwanted axial harmonics are integrated out and yet a profitably large portion of the fundamental is spanned (Fig. 11.2).

The question of pulsing the large subcritical facility of the MITR under the restrictive conditions and the proximity of the operating reactor pose special problems, and considerable attention must be directed to minimizing the neutron and gamma-ray background from the reactor and the lattice and maximizing the beam current on the target and therefore the neutron source strength

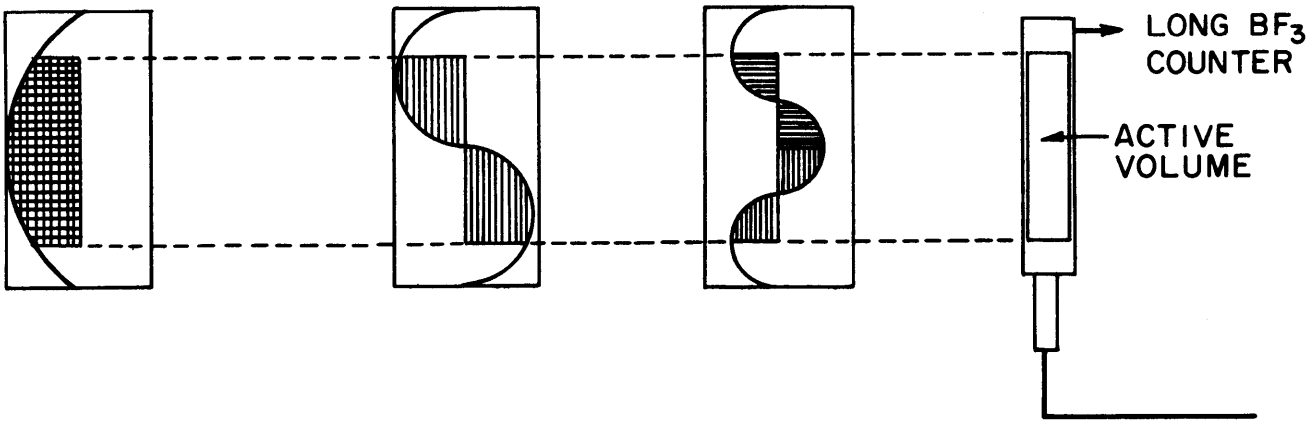

FIG. II.2 USE OF LONG BF_3 COUNTERS TO SUPPRESS THE UNWANTED AXIAL HARMONICS

available. These considerations have prompted a detailed investigation of the intensity requirements on the source which would give a reasonable experimental run time using a typical well-sized detector and duty cycle.

The total thermal neutrons population at a time t, after the burst, is obtained, in terms of its modal spectrum, by integrating the time-dependent neutron density, n(r, t), over the volume, v, of the assembly. The result is:

$$N(t) = \int_v n(\underline{r})\, d\underline{r} = p \sum_m \sum_n S_{mn} \frac{2R^2H}{a_m} J_1(a_m) \left\{\frac{1-(-1)^n}{n}\right\} e^{-B^2_{mn}\tau}\, e^{-\lambda_{mn}t} \tag{11.9}$$

Taking, for illustration, the source to be at the center, so that

$$S_{mn} = \frac{2S_o}{\pi R^2 H[J_1(a_m)]^2} \sin\frac{n\pi}{2},$$

we find the number of neutrons in the fundamental (m = 1 = n) mode at time t > o:

$$N_{11}(t) = p\frac{8}{\pi}\frac{S_o}{a_1 J_1(a_1)}\, e^{-B^2\tau}\, e^{-\lambda_{11}t} \tag{11.10}$$

A fraction, $\frac{L^2B^2}{1+L^2B^2}$, of these leak out of the assembly; by putting in typical values for the parameters, it is found that of the δS_o neutrons in each burst of width δ sec, about 25% eventually leak out in the fundamental mode to be counted. By choosing reasonable values for the over-all detection efficiency, repetition rate, pulse width, etc., it is estimated that the source intensity must be of the order of 10^6-10^7 n/burst in order that there may be accumulated in the first analysis channel about 10^5 counts in a total running time of around an hour.

The typical meanlife in the lattices to be investigated is about 2 msec. It is expected that for monitoring the decay over four decades in heavy water lattices, workable values would be below 50 pps for the rep. rate and about 10 μsec for the pulse width.

11.3 CONTROL ROD THEORY

The results of the control rod studies are to be compared with theoretical evaluations of control rod effects. The literature of control rod theory has been reviewed from this standpoint. No new theoretical results have, as yet, been obtained.

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